- **Complex periodic waves**
- **Components and spectra**

Background reading:

- *AAP* Ch 1, sec 1.3.2

Today's plan

- Quick discussion: Lab #1
- Complex periodic waves
- How sine waves add to produce complex waves
- How a spectrum represents a complex wave
- If you have prior background in acoustics or high-school physics, some of this may be review for you
  - But remember: The next few classes will build on one another quickly, so **be sure you are solid on these basics** before we go on

0. Lab #1

- Resources for connecting symbols to sounds
- Did you follow the instructions? Did you read the assigned Praat handouts #1-4 before the lab?
- The screenshot question
  - Zooming in to the right view
  - Playing a sound file / parts of a sound file
  - Highlighting the right area
  - Taking a screenshot
- Any other comments or questions?

1. Complex periodic waves

- Review: A **complex periodic** wave
    - Why **periodic**? →
    - Why **complex**? →

1. Complex periodic waves

- Review: A **complex periodic** wave
  - Why **periodic**? → has a repeating waveform
  - Why **complex**? → is not a sine wave

- What did you observe about musical notes [1] and [2] in the prep questions for today's class?
  - Simple or complex?
  - Which had the higher frequency?
  - Which had the higher average amplitude?

1. Complex periodic waves

- Any **complex periodic wave** can be mathematically analyzed as a combination of simple periodic waves
  - The simple waves that combine to form a complex wave are its **components**
  - The process of finding the components of a complex wave is known as Fourier analysis; Praat does this for us
- Therefore, we can **define** (describe) any complex wave by **listing its components**
  - Each component (sine wave) can be defined by its **amplitude** and **frequency** (phase doesn't have much effect in speech analysis)

1. Complex periodic waves

- Example: *AAP*, Ch 1, Figure 1.6 (p 15)
  - How are the first four waveforms related to the fifth?

1. Complex periodic waves

- The **fundamental frequency** (***f_0***) of a complex periodic wave is the frequency with which the **whole complex pattern** itself repeats

- The fundamental frequency of a complex wave can be calculated:

  → It is the **greatest common *divisor* (GCD)** of the frequencies of its components

  - **Error** in *AAP*, p 13: Johnson says "greatest common *denominator*," but this is wrong!

1. Complex periodic waves

- The fundamental frequency of a complex wave can be calculated:

  → It is the **greatest common divisor (GCD)** of the frequencies of its components

- Example: Suppose a complex wave has components at 200 Hz, 300 Hz, and 500 Hz

  - GCD (largest number that divides all evenly) = 100 Hz
  - The fundamental frequency of this complex wave is thus 100 Hz, ***even though it has no component at 100 Hz*** (we will test this shortly)

2. Adding sine waves

- Let's look at how **adding sine waves** together produces a complex wave:
  - Visit the web page "Wave Adder" (from Zona Land)
  - This interface lets you input the amplitude (**A**) and frequency (***f***) of individual sine waves, and add them together

2. Adding sine waves

- Using the "Wave Adder": How it works
  - The black box is where the waves will appear
  - The red line indicates one unit of **time** (say, 1 s)
  - To **draw** a sine wave, enter values in the boxes for **amplitude** and **frequency** and then click "Draw yellow" | We won't be using phase shift yet, and we'll keep the 'sine' box checked — you can change these settings on your own if you are curious
  - To **add** the new wave to the previous combination, click "Add yellow to white"
  - To **start over**, click "Reset"

2. Adding sine waves

- Try it out: Using the "Wave Adder" to see what happens when sine waves are added
  1. When you start the Wave Adder, the "Amplitude" box shows 10 and the "Frequency" box shows 1 — keep those values, A=10 and f=1
  2. Click "Draw yellow" to see this sine wave
  3. Click "Add yellow to white"; the wave changes to white, meaning it's added to the combination (since there are no other waves yet, the white wave has the same shape as the yellow one did)

2. Adding sine waves

4. Draw a new wave with the values A=2 and *f*=8
   (put the values in the boxes and click on "Draw yellow")
5. Before you add the waves together, take a look:
   What should happen when these waves add?

2. Adding sine waves

4. Draw a new wave with the values A=2 and *f*=8 (put the values in the boxes and click on "Draw yellow")
5. Before you add the waves together, take a look:
   - At some time points, both the "big slow" wave and the "small fast" wave have displacement in the **same** direction, positive (up) or negative (down): these effects will **combine**
   - At other time points, the waves have displacement in **opposite** directions — these effects will (partially) **cancel** each other out
6. Click "Add yellow to white" — Can you see how each sine wave contributes to the complex wave?

2. Adding sine waves

- Using the "Wave adder": Testing the GCD claim
  - Draw a sine wave with A=10 and *f*=2; add it
  - Draw a sine wave with A=8 and *f*=3; add it
  - Draw a sine wave with A=2 and *f*=5; add it
- What is the GCD of 2, 3, and 5 Hz? → GCD = **1 Hz**
- What is the fundamental frequency of the complex wave you have drawn?

  → The wave repeats **one** time by the red line (= 1 s)
- This shows that a complex wave doesn't have to have **components** at its fundamental frequency!

2. Adding sine waves

- Using the "Wave adder": Replicating *AAP* Fig 1.6 (p 15)
  - Draw a sine wave with A=10 and *f*=1; add it
  - Draw a sine wave with A=5 and *f*=2; add it
  - Draw a sine wave with A=3.3 and *f*=3; add it
  - Draw a sine wave with A=2.5 and *f*=4; add it
  - Draw a sine wave with A=2 and *f*=5; add it

  How does your result compare with *AAP* Fig 1.6?

  (Fig 1.6 details: *AAP*, p 16; to make this work on the Wave Adder, amplitudes are multiplied by 10 and frequencies divided by 100)

- In just this way, **any** complex wave can be represented as a combination of sine waves

2. Adding sine waves

- Here is another example of adding sine waves together: see the web page "Standing Waves, Medium Fixed at Both Ends" (from Zona Land)
  - The complex wave is shown in white
  - The component sine waves (called "harmonics" here; we'll learn that term later) are shown in other colors
  - You can add and remove components from the complex wave by clicking the check-boxes — try it and observe what effect this has

3. Spectra

- Look at Figures 1.6 and 1.7 in *AAP* (pp 15, 16)
  - How is Fig 1.6 related to Fig 1.7?

3. Spectra

- Look at Figures 1.6 and 1.7 in *AAP* (pp 15, 16)
  - How is Fig 1.6 related to Fig 1.7?

- What information do we need to state in order to describe a **complex wave**?

3. Spectra

- Look at Figures 1.6 and 1.7 in *AAP* (pp 15, 16)
    - How is Fig 1.6 related to Fig 1.7?

- What information do we need to state in order to describe a **complex wave**? → List its components

- What kind of waves are the components of a complex wave?

3. Spectra

- Look at Figures 1.6 and 1.7 in *AAP* (pp 15, 16)
  - How is Fig 1.6 related to Fig 1.7?

- What information do we need to state in order to describe a **complex wave**? → List its components

- What kind of waves are the **components** of a complex wave? → Sine waves

- What information do we need to state in order to describe a **sine wave**?

3. Spectra

- Look at Figures 1.6 and 1.7 in *AAP* (pp 15, 16)
  - How is Fig 1.6 related to Fig 1.7?

- What information do we need to state in order to describe a **complex wave**? → List its components

- What kind of waves are the **components** of a complex wave? → Sine waves

- What information do we need to state in order to describe a **sine wave**? → **frequency** and **amplitude**

  (technically also phase, but we don’t really need this for speech analysis)

3. Spectra

- The **spectrum** of a wave is a graphic representing the amplitude and frequency of each component
  - Also called **line spectrum**, **power spectrum**, or (in Praat) **spectral slice**
- The graphic in *AAP* Fig 1.7 is the spectrum of the complex wave in Fig 1.6
- A sine wave has a spectrum, too — how many lines are on it?

3. Spectra

- The **spectrum** of a wave is a graphic representing the amplitude and frequency of each component
  - Also called **line spectrum**, **power spectrum**, or (in Praat) **spectral slice**

- The graphic in *AAP* Fig 1.7 is the spectrum of the complex wave in Fig 1.6

- A sine wave has a spectrum, too — how many lines are on it?

  → One line, for one sine-wave component

3. Spectra

- What property is represented on each **axis** of a spectrum display?

3. Spectra

- What property is represented on each **axis** of a spectrum display?
  - **Amplitude** on the y axis (vertical) and **frequency** on the x axis (horizontal)
  - Note that a spectrum does **not** represent **time**

3. Spectra

- Let’s use Praat to view a spectrum (“spectral slice”) of note [2] from today’s prep questions
  - Simple or complex wave?
  - Can we use the spectrum to determine the f_0?

- You will work with spectral slices in Praat in Lab #2