- **Standing waves**
- **Resonances**

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*Background reading and web activities:*

- *AAP* Ch 2, sec 2.1, second half
- "…Standing Wave Diagrams 1" (Zona Land)
  "Wave Interference 2" (Zona Land)

# 0. Today's plan

- Standing waves
- Resonances
- Boundary conditions
  - On a string
  - In a tube
- Calculating resonance wavelengths

# 0. Today's plan

In the discussion this week, we will **emphasize the concepts** before introducing the formulas.

You can remember (or reinvent) the formulas more easily if you understand the concepts!

Upcoming labs will give you a chance to practice **applying** the concepts (and the formulas).

# 1. Standing waves

- Why does a **real-world object** vibrate in a way that produces **complex waves**?
  - Objects have **multiple modes of vibration**
  - This is because multiple waves “fit” an object

# 1. Standing waves

- Why does a **real-world object** vibrate in a way that produces **complex waves**?
  - Objects have **multiple modes of vibration**
  - This is because multiple waves “fit” an object
- “Violin string” demo, Zona Land (also seen last week)
  - String vibrates with “one loop”, “two loops”, etc.
  - Each vibration pattern → **one sine wave**
  - When **multiple** vibration patterns are present, their sine waves are **added together**
    → *What do we get when sine waves are added?*

# 1. Standing waves

- What “size” waves will “fit” a vibrating object?
  - A wave that “fits” a particular object is called a **resonance** of that object
  - A resonance forms a **standing wave**:  an oscillating pattern, stable in space over time

- What does a standing wave look like?
  See web demo “Wave Interference 2” (Zona Land)
  - Setting “Sinusoid 6” creates a standing wave
  - Compare “Sinusoid 8”:  not a standing wave

# 1. Standing waves

- Standing waves arise because of **reflection** and **interference**
- We will not pursue this in detail; think of it this way:
  - When a “right-size wave” *reflects* from the edge of the object, the outgoing and returning waves *interfere* in a way that makes a standing wave (stable oscillation)
  - When a “wrong-size wave” *reflects* from the edge of the object, the outgoing and returning waves *interfere* in a way that is not a stable oscillation

# 1. Standing waves

- *Warning:* **Standing-wave diagrams** are graphs showing **amplitude** by **distance**
  - This is like the snapshot of the waves on the surface of a lake, showing water height at different physical locations
  - This is ***not*** like a typical waveform plot of a sound wave, which plots amplitude by **time** at a fixed location
- *This is useful!* The resonances of a tube of air (like the vocal tract!) depend on its **physical length**, which we can measure or calculate

# 1. Standing waves

- Some key terminology:
  - **node** — physical position on standing wave that **always has zero amplitude**

    = location in space where a wave and its reflection always *cancel each other out*

  - **antinode** — physical position on standing wave with **maximum amplitude change from zero**

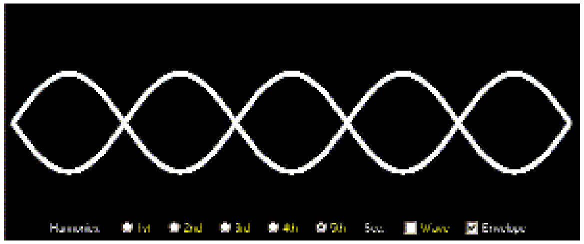
    (includes both **positive and negative** extreme values)

    = location in space where a wave and its reflection *maximally reinforce each other*

# 1. Standing waves

- **Standing-wave diagrams** typically show the **envelope** of the standing wave
  - This shows the *maximum* amplitude reached at each physical position along the wave
  - See web demo "Understanding Standing Wave Diagrams 1" (Zona Land)

- Can you identify the **nodes** and **antinodes** on a standing-wave diagram?



(from the demo linked above)

# 2. Resonances

*So far, we've considered these ideas:*

- Why does a **real-world object** vibrate in a way that produces **complex waves**?
  - Objects have **multiple modes of vibration**
  - **Why?**

# 2. Resonances

*So far, we've considered these ideas:*

- Why does a **real-world object** vibrate in a way that produces **complex waves**?
  - Objects have **multiple modes of vibration**
  - **This is because multiple waves "fit" an object**

*What "size" waves will "fit" a vibrating object?*

# 2. Resonances

*So far, we've considered these ideas:*

- Why does a **real-world object** vibrate in a way that produces **complex waves**?
  - Objects have **multiple modes of vibration**
  - This is because **multiple waves "fit" an object**

*What "size" waves will "fit" a vibrating object?*

- A wave "fits" if it forms a **standing wave**: an oscillating pattern, stable in space over time
  - Such a wave is called a <u>**resonance**</u> of that object

# 2. Resonances

- Next, we want to be able to **determine** what the **resonances** ("waves that fit") **of a particular object** actually are
  - We will talk about **strings** first (easy to visualize)
  - But our main focus will be on **air in a tube**

- *Preview: **Speech-sound analyses** that depend on resonance frequencies of **air in a tube** will include:*
  - *Vowel formants (indicate height, backness, rounding)*
  - *Consonant place of articulation (affects vowel formants)*
  - *Fricative noise spectra / stop burst spectra*
  - *Acoustic signatures of nasals and laterals*

# 2. Resonances

- We can model the **multiple modes of vibration** of a **string**, or of **air in a tube**
  - To do this, we determine the **wavelength** of each of the **resonances** of the system, based on the physical size of the system
  - Then (for air in a tube) we can calculate the **frequency** of each of the resonances

# 2. Resonances

- The "compatible waves" ("waves that fit") for a string or a tube are determined by
  - its **length**
  - its **boundary conditions**

- **Boundary conditions**:  For each **end** of the string or tube, is it a **node** or an **antinode**?
  - String:  Is the end ***fixed*** (**node**) or ***free*** (**antinode**)?
  - Tube:  Is the end ***open*** (**node** of pressure wave) or ***closed*** (**antinode** of pressure wave)?

# 3. Boundary conditions (1): Node/node

- A **string**, fixed at both ends: **Node/node system**
  - At a point where a string is **fixed**, its displacement *can only be zero* = **node**
  - If **both ends** of the string are **fixed**, what are the "compatible waves" for this system?

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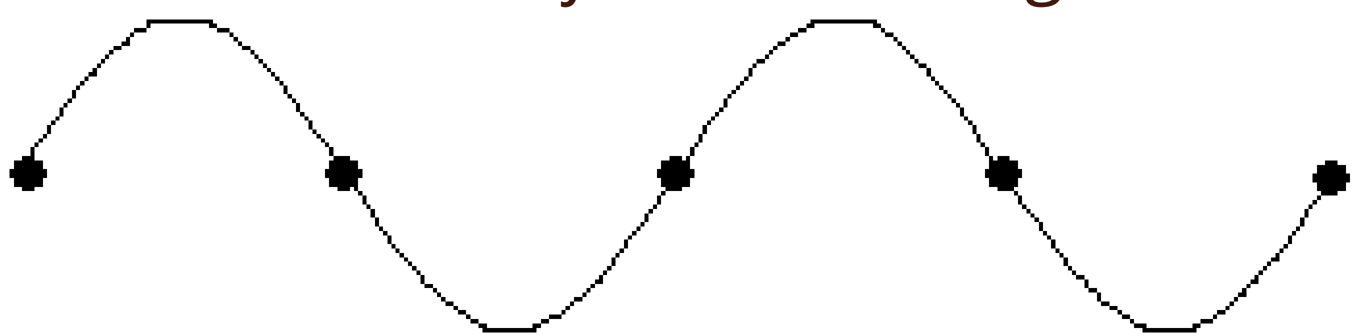
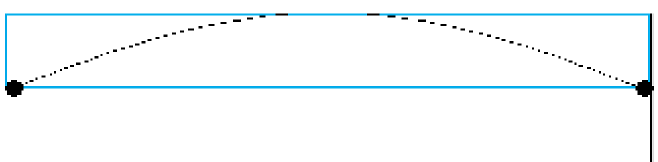
*"How much wave" can fit on the string?*



# 3. Boundary conditions (1): Node/node

- A **string**, fixed at both ends: **Node/node system**
  - At a point where a string is **fixed**, its displacement *can only be zero* = **node**
  - If **both ends** of the string are **fixed**, what are the "compatible waves" for this system?

- Here is the **first** resonance (*longest* wavelength):

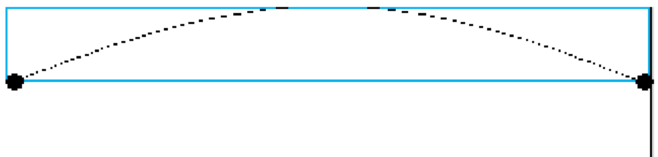


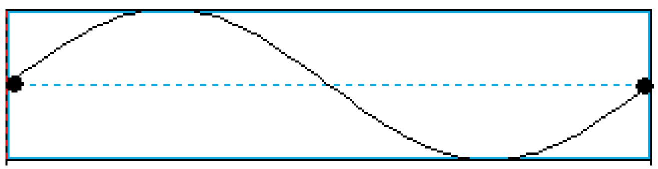
→ ½ *cycle* of the wave "fits"

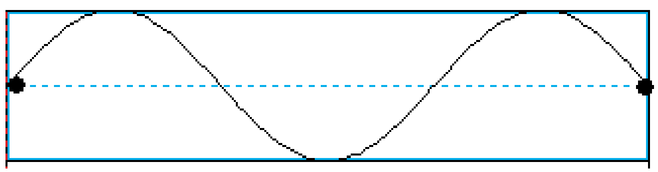
What are the rest?

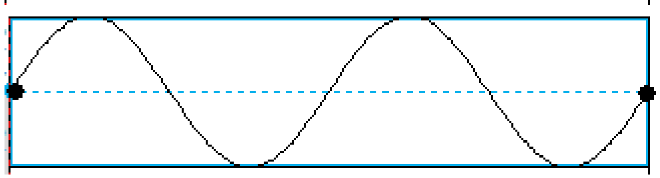
# 3. Boundary conditions (1): Node/node

- A **string**, fixed at both ends: **Node/node system**, also called a **half-wavelength system** (why?)
- Here are the first four **resonances**

*1st*  → *½ cycle* of the wave "fits"

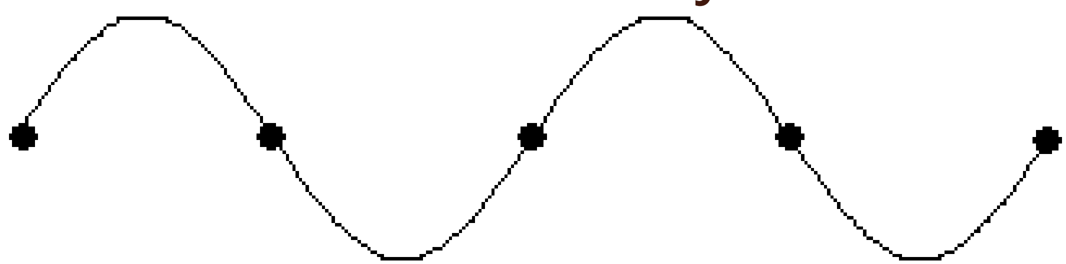
*2nd*  → *1 cycle* of the wave "fits"

*3rd*  → 1*½ cycles* of the wave "fit"

*4th*  → *2 cycles* of the wave "fit"

# 3. Boundary conditions (1): Node/node

- A **tube**, open at both ends: **Node/node system**
  - At a point where a tube is **open**, the air pressure interfaces with the outside world, so the pressure displacement *can only be zero* = **node**
  - If **both ends** of the tube are **open**...
    *"How much wave" can fit in the tube?*

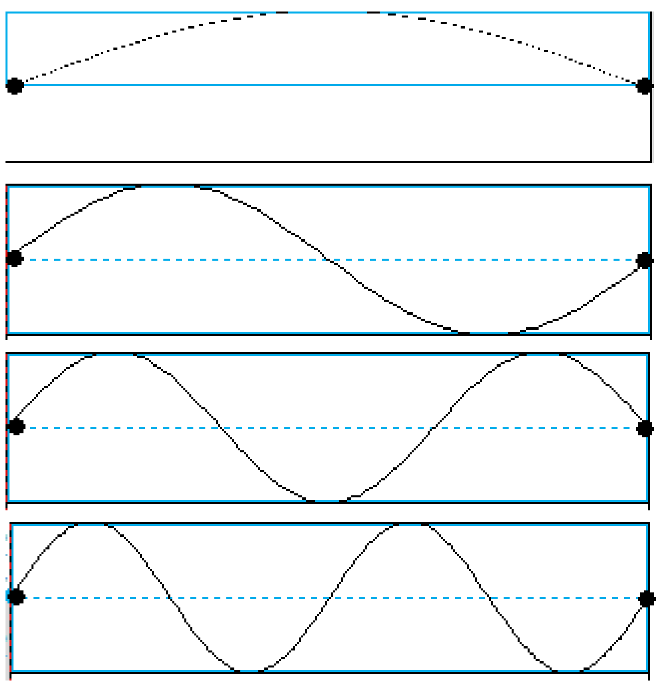


- See web demo "Standing Sound Waves" for more about pressure waves in a tube and standing-wave diagrams — the **red graph** in that demo shows the **pressure** wave

# 3. Boundary conditions (1): Node/node

- A **tube**, open at both ends: **Node/node system**, also called a **half-wavelength system**
  - Exactly like the string case discussed above!

- Here are the first four **resonances**



*1st* → ½ *cycle* of the wave “fits”

*2nd* → *1 cycle* of the wave “fits”

*3rd* → *1½ cycles* of the wave “fit”

*4th* → *2 cycles* of the wave “fit”

# 3. Boundary conditions (2): Node/antinode

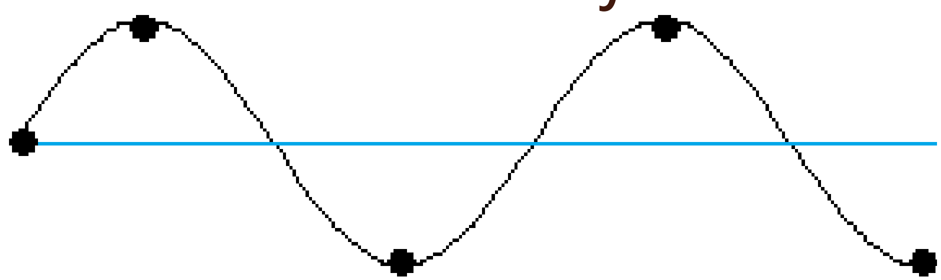
- A **tube**, open at one end and closed at the other: **Node/antinode system**
  - **Open** end: A pressure-wave **node** (see above)
  - **Closed** end: The reflecting waves will create a region of *maximum compression* alternating with *maximum rarefaction* → pressure-wave **antinode**

- *Optional:* For more about the reflection of pressure waves in tubes, see web demo "Animations of sound waves in open and closed tubes" (UNSW)
  - What happens at the edges when the wave reflects?

# 3. Boundary conditions (2): Node/antinode

- A **tube**, open at one end and closed at the other: **Node/antinode system**
  - **Open** end: A pressure-wave **node**
  - **Closed** end: A pressure-wave **antinode**

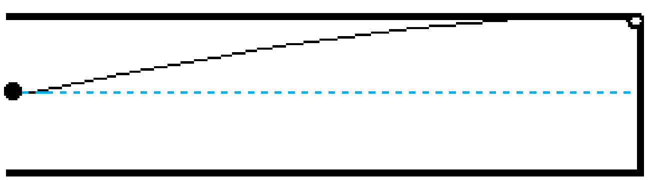
*"How much wave" can fit in the tube?*



# 3. Boundary conditions (2): Node/antinode

- A **tube**, open at one end and closed at the other: **Node/antinode system**
  - **Open** end: A pressure-wave **node**
  - **Closed** end: A pressure-wave **antinode**

- Here is the **first** resonance (*longest* wavelength):



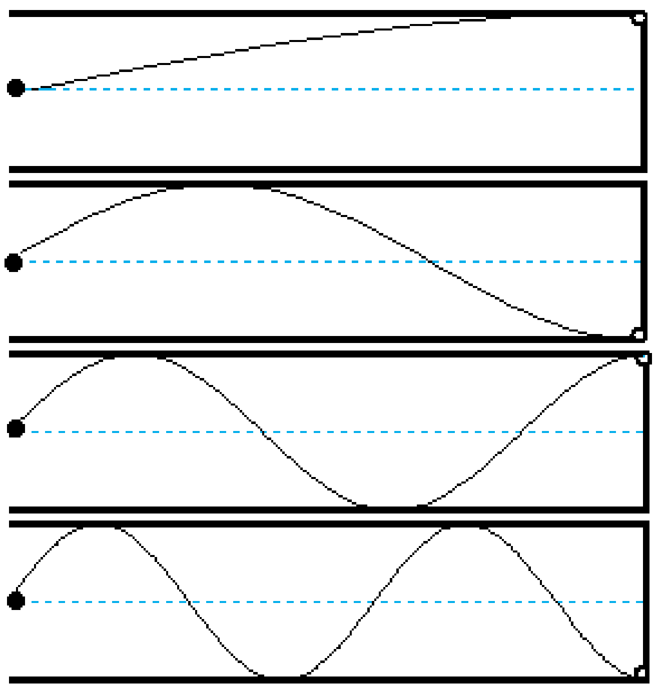
→ ¼ *cycle* of the wave "fits"

What are the rest?

# 3. Boundary conditions (2): Node/antinode

- A **tube**, open at one end and closed at the other:
  **Node/antinode system**,
  also called a **quarter-wavelength system** (why?)

- Here are the first four **resonances**



*1st* → *¼ cycle* of the wave “fits”

*2nd* → *¾ cycle* of the wave “fits”

*3rd* → *1¼ cycles* of the wave “fit”

*4th* → *1¾ cycles* of the wave “fit”

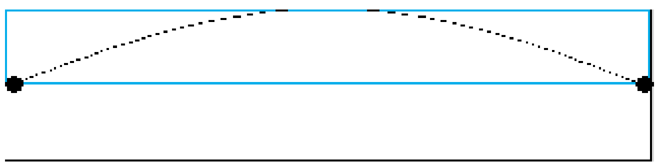
# 4. Calculating resonance wavelengths

- If we know
  - the **length** of the string or tube (***L***)
  - **"how much wave" fits** on the string or tube for the ***n*th resonance**

- We can calculate the **wavelength $\lambda_n$** for the ***n*th resonance**

# 4. Calculating resonance wavelengths

- Consider the **first resonance** of a **node/node** (half-wavelength) system

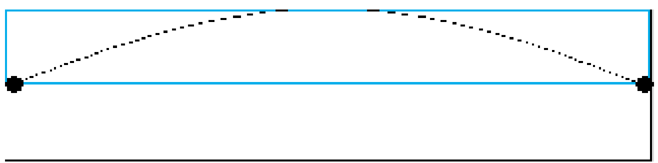


→ ½ *cycle* of the wave "fits"

- If the string or tube is length *L*,
  what is the **wavelength** of the first resonance ($\lambda_1$)?

# 4. Calculating resonance wavelengths

- Consider the **first resonance** of a **node/node** (half-wavelength) system



→ ½ *cycle* of the wave "fits"

- If the string or tube is length *L*,
  what is the **wavelength** of the first resonance ($\lambda_1$)?

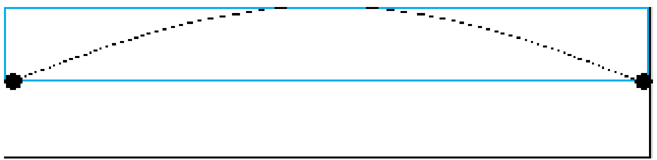
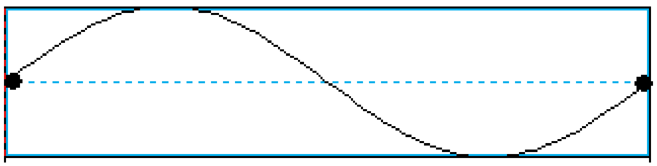
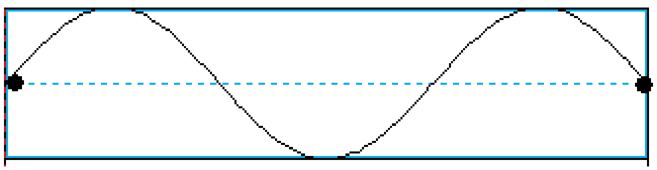
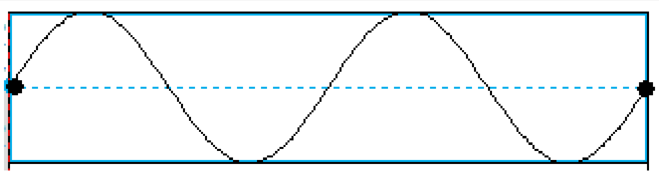
*L* fits half of the wave

*L* is half as long as $\lambda_1$

$\boldsymbol{\lambda_1 = 2L}$

# 4. Calculating resonance wavelengths

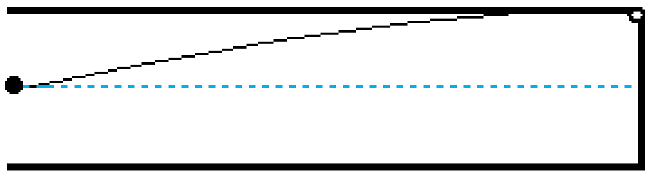
- **Node/node** (half-wavelength) system
  Tube or string of length *L*

| | | | |
|---|---|---|---|
|  | $L = ½ \cdot \lambda_1$ | $\lambda_1 = 2\ L$ | $\lambda_1 = (2/1) \cdot L$ |
|  | $L = 1 \cdot \lambda_2$ | $\lambda_2 = L$ | $\lambda_2 = (2/2) \cdot L$ |
|  | $L = 1½ \cdot \lambda_3$ | $\lambda_3 = ⅔\ L$ | $\lambda_3 = (2/3) \cdot L$ |
|  | $L = 2 \cdot \lambda_4$ | $\lambda_4 = ½\ L$ | $\lambda_4 = (2/4) \cdot L$ |

- ***General formula:*** $\boldsymbol{\lambda_n = (2/n)\ L}$ or $\boldsymbol{\lambda_n = 2L/n}$

# 4. Calculating resonance wavelengths

- Consider the **first resonance** of a **node/antinode** (quarter-wavelength) system

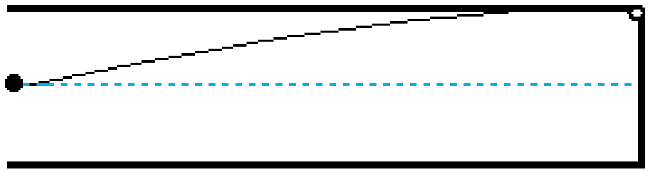


→ ¼ *cycle* of the wave “fits”

- If the tube is length *L*,
  what is the **wavelength** of the first resonance ($\lambda_1$)?

# 4. Calculating resonance wavelengths

- Consider the **first resonance** of a **node/antinode** (quarter-wavelength) system



→ ¼ *cycle* of the wave "fits"

- If the tube is length *L*,
  what is the **wavelength** of the first resonance ($\lambda_1$)?

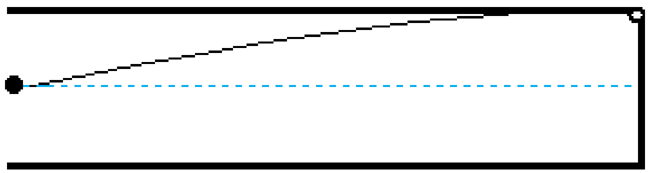
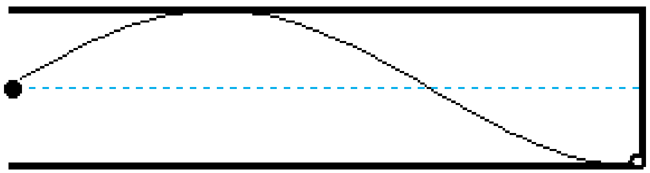
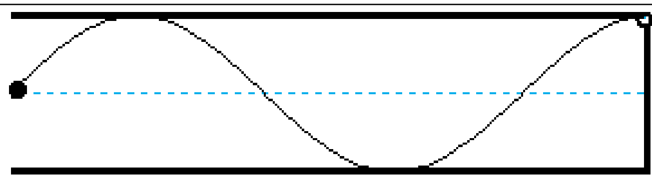
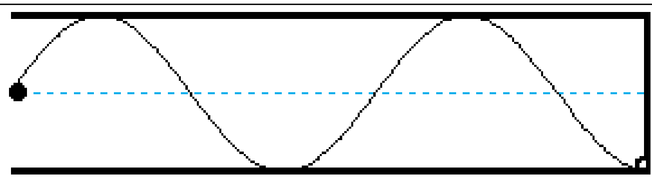
*L* fits one quarter of the wave

*L* is one quarter as long as $\lambda_1$

$$\boldsymbol{\lambda_1 = 4L}$$

# 4. Calculating resonance wavelengths

- **Node/antinode** (quarter-wavelength) system
  Tube of length $L$

| | | | |
|---|---|---|---|
|  | $L = \frac{1}{4} \cdot \lambda_1$ | $\lambda_1 = 4\,L$ | $\lambda_1 = (4/1) \cdot L$ |
|  | $L = \frac{3}{4} \cdot \lambda_2$ | $\lambda_2 = 4/3\,L$ | $\lambda_2 = (4/3) \cdot L$ |
|  | $L = 1\frac{1}{4} \cdot \lambda_3$ | $\lambda_3 = 4/5\,L$ | $\lambda_3 = (4/5) \cdot L$ |
|  | $L = 1\frac{3}{4} \cdot \lambda_4$ | $\lambda_4 = 4/7\,L$ | $\lambda_4 = (4/7) \cdot L$ |

- ***General formula:*** $\lambda_n = (4/(2n-1)) \cdot L$ or $\lambda_n = 4L / (2n-1)$

# 5. Calculating resonance frequencies

- Finally!—the **frequencies** of the resonances are what we really want to know
  - Knowing the resonance frequencies of a tube of air (in the vocal tract) helps us **model the acoustics of speech sounds**
- *Example:* Soon we will learn about the **source-filter model** of speech acoustics as applied to **vowels**
  - The **source** is the glottal-source wave
  - The **filter** is determined by the **resonance frequencies of the vocal tract**

# 5. Calculating resonance frequencies

- **Wavelength (λ)** and **frequency (*f*)** are related:

$$c=\lambda f$$

  - where ***c*** is the speed of the wave (about **350 m/s** for sound in air, according to *AAP*)

- Wavelength and frequency are ***inversely*** related
  - **Long** wavelength means **low** frequency
  - **Short** wavelength means **high** frequency

Imagine traffic moving by at a steady 35 mph. Many VW bugs (short) would go by in 1 minute (higher frequency), but few buses (long) would go by in 1 minute (lower frequency).

# 5. Calculating resonance frequencies

- If we know wavelength, we can solve for frequency

  $c=\lambda f$

  $\boldsymbol{f=c/\lambda}$

- Find the **frequency** of the ***n*th resonance (*$f_n$*)**:
  - Plug the wavelength $\lambda_n$ into the formula
  - Solve for $f_n$

# 5. Calculating resonance frequencies

- For a **node/node** system with tube of length $L$

| | |
|---|---|
| $\lambda_n = 2L/n$ | relates wavelength to tube length |
| $f_n = c/\lambda_n$ | relates frequency to wavelength |
| $f_n = c \,/\, (2L/n)$ | relates frequency to tube length |
| $\boldsymbol{f_n = n \bullet c/2L}$ | |

- **Shortcut!** Once you know the **1st resonance $f_1$** :

$$f_n = n \bullet f_1$$

→ The resonance frequencies in a **node/node** system are **whole-number multiples** of $f_1$

# 5. Calculating resonance frequencies

- For a **node/antinode** system with tube of length *L*

| | |
|---|---|
| $\lambda_n = 4L / (2n-1)$ | relates wavelength to tube length |
| $f_n = c/\lambda_n$ | relates frequency to wavelength |
| $f_n = c / (4L / (2n-1))$ | relates frequency to tube length |
| $\mathbf{f_n = (2n-1) \cdot c / 4L}$ | |

- **Shortcut!** Once you know the **1st resonance $f_1$** :

$$\mathbf{f_n = (2n-1) \cdot f_1}$$

→ The resonance frequencies in a **node/antinode** system are **odd-numbered multiples** of $f_1$