

- **Resonance frequencies**
- **The glottal source**

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*Background reading:*

- *AAP* Ch 2, sec 2.4
- *AAP* Ch 2, sec 2.1, first half

# 0. Today's plan

- Review/check-in
  - Standing waves
  - Resonances
  - Boundary conditions (string, tube)
  - Calculating resonance wavelengths
- Calculating resonance frequencies
- The glottal source

# 0. Today's plan

In the discussion this week, we will **emphasize the concepts** before introducing the formulas.

You can remember (or reinvent) the formulas more easily if you understand the concepts!

Upcoming labs will give you a chance to practice **applying** the concepts (and the formulas).

# 1. Review: Standing waves, resonances

- What is the connection between **standing waves** and **resonances**?

# 1. Review: Standing waves, resonances

- What is the connection between **standing waves** and **resonances**?
  - A wave that “fits” a particular object is called a **resonance** of that object
  - A resonance forms a **standing wave**: an oscillating pattern, stable in space over time
- Which of these settings show a standing wave?  
See web demo [“Wave Interference 2”](#) (Zona Land)
  - “Sinusoid 4” | “Sinusoid 5” | “Sinusoid 10”

# 1. Review: Standing waves, resonances

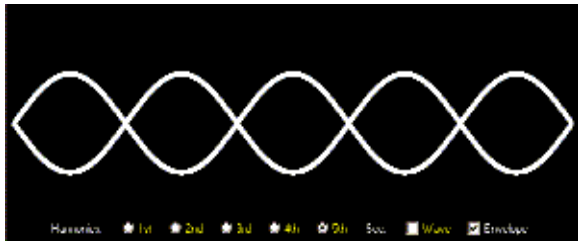
- What are the boundary conditions for...
  - The fixed end of a string
  - The free end of a string
  - The open end of a tube of vibrating air
  - The closed end of a tube of vibrating air
- **Why?**

# 1. Review: Standing waves, resonances

- What are the boundary conditions for...
  - The fixed end of a string | node
  - The free end of a string | antinode
  - The open end of a tube of vibrating air  
| (pressure) node / (displacement) antinode
  - The closed end of a tube of vibrating air  
| (pressure) antinode / (displacement) node
- **Why?** | physical conditions determine this!

# 1. Review: Standing waves, resonances

- If this is a standing wave on a string of length 10cm, what is the wavelength of the standing wave?



(from Zona Land [Standing Waves demo](#))

- Checking in: What are the measurement units of the wavelength, and why?
- How can we calculate the wavelength?



# 1. Review: Standing waves, resonances

- We can model the **multiple modes of vibration** of a **string**, or of **air in a tube**
  - To do this, we **determine the wavelength** of each of the **resonances** of the system, based on the physical size of the system
  - Then (for air in a tube) we can **calculate the frequency** of each of the resonances

# 1. Review: Standing waves, resonances

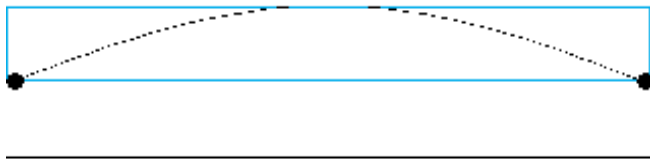
- The resonances (“waves that fit”) for a string or a tube are determined by
  - its **length**
  - its **boundary conditions**

# 1. Review: Standing waves, resonances

- A **string**, fixed at both ends
  - What are the **boundary conditions**?
  - What does the standing-wave diagram for the first resonance look like? **Why?**

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node/node
  - What does the standing-wave diagram for the first resonance look like? **Why?**



$L$

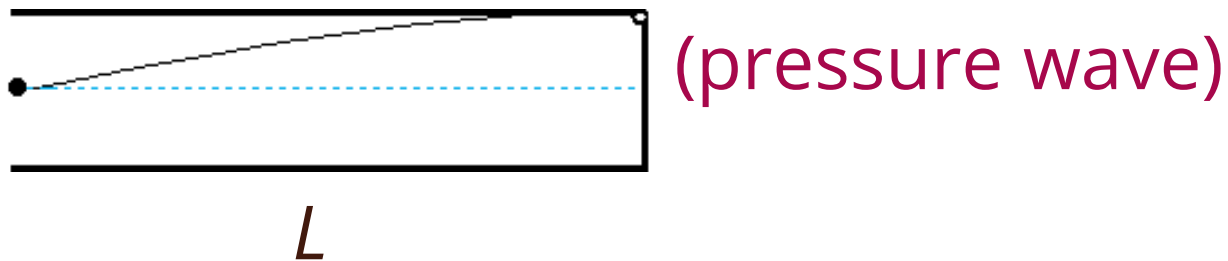
- What is the wavelength of this resonance, for a string of length  $L$ ? **Why?**

# 1. Review: Standing waves, resonances

- A **tube**, open at one end and closed at the other
  - What are the **boundary conditions**?
  - What does the standing-wave diagram for the first resonance look like? **Why?**

# 1. Review: Standing waves, resonances

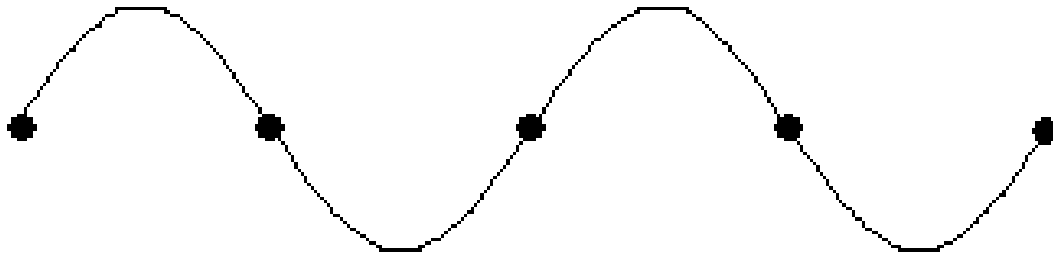
- A **tube**, open at one end and closed at the other
  - What are the **boundary conditions**?  
(pressure) node/antinode | (displacement) A/N
  - What does the standing-wave diagram for the first resonance look like? **Why?**



- What is the wavelength of this resonance, for a tube of length  $L$ ? **Why?**

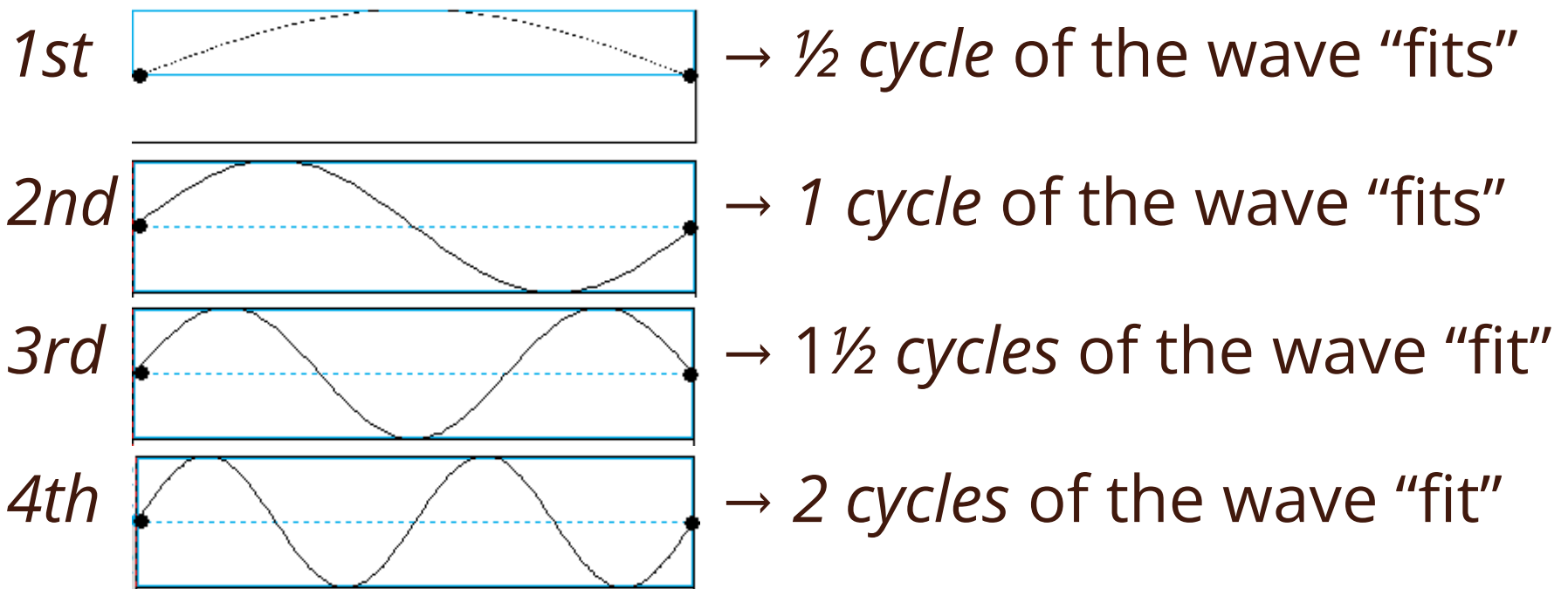
# 1. Review: Standing waves, resonances

- A **string**, fixed at both ends: **Node/node system**
- What are the **first four** resonances?
  - Hint: Think about the boundary conditions



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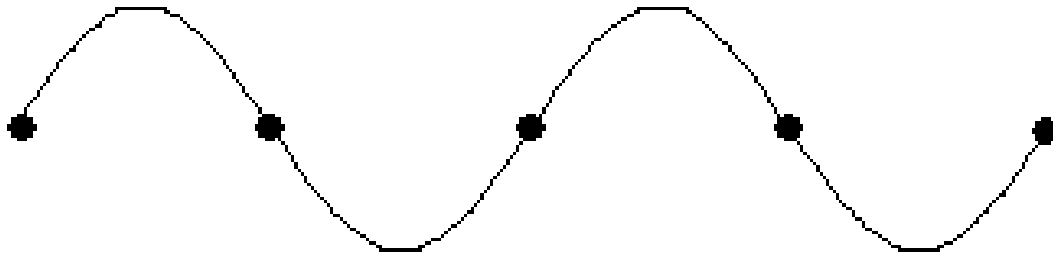


- Why is this called a **half-wavelength system**?



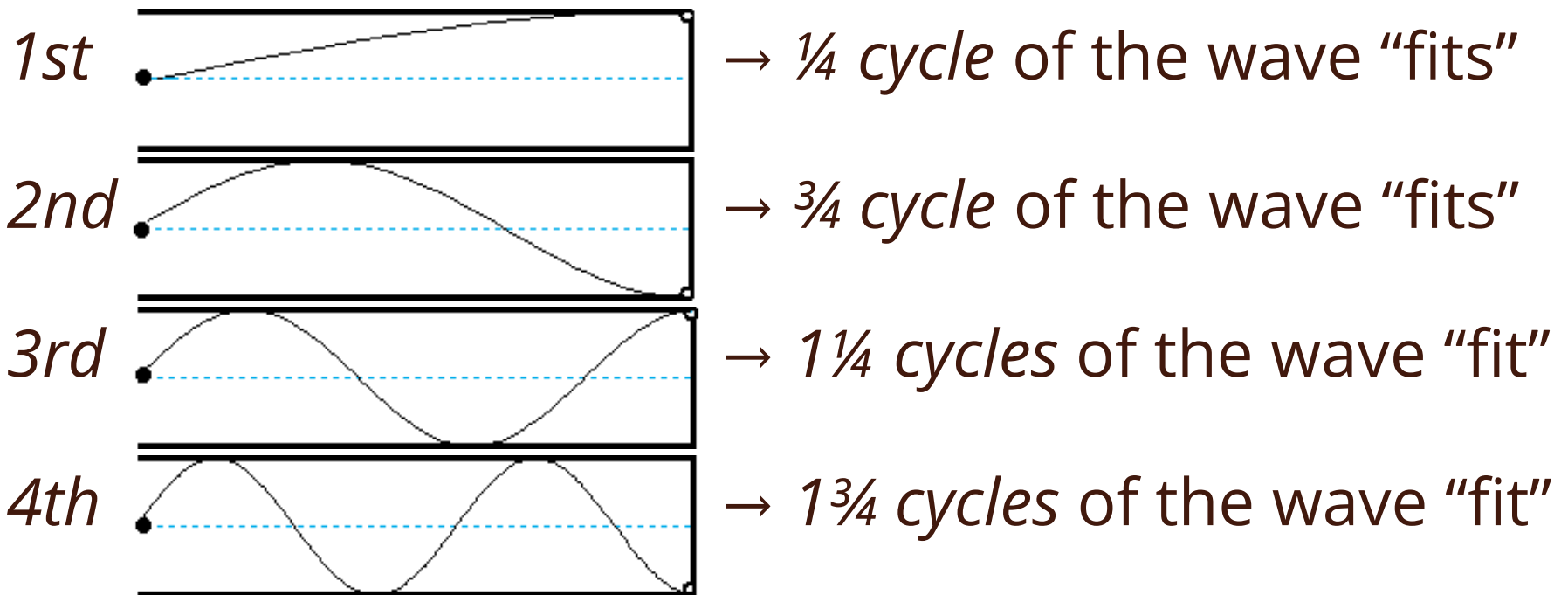
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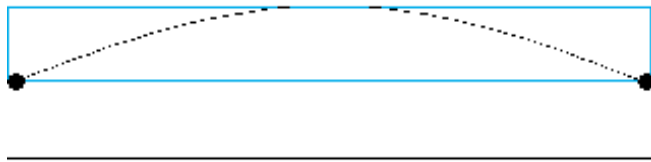
- Why is this called a **quarter-wavelength system**?

## 2. Calculating resonance wavelengths

- If we know
  - the **length** of the string or tube ( $L$ )
  - “**how much wave**” fits on the string or tube for the  **$n$ th resonance**
- We can calculate the **wavelength  $\lambda_n$**  for the  **$n$ th resonance**

## 2. Calculating resonance wavelengths

- Consider the **first resonance** of a **node/node** (half-wavelength) system

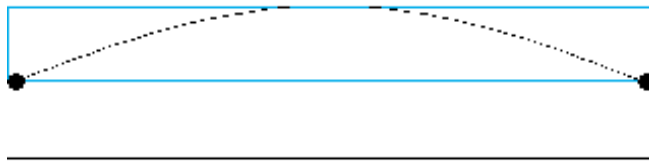


→  $\frac{1}{2}$  cycle of the wave "fits"

- If the string or tube is length  $L$ , what is the **wavelength** of the first resonance ( $\lambda_1$ )?

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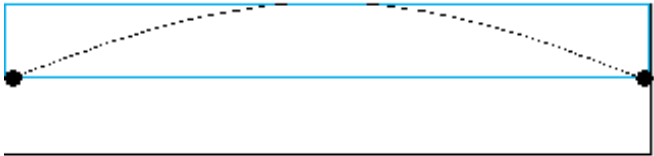
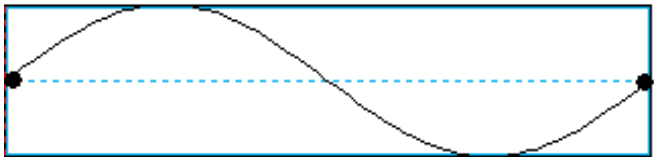
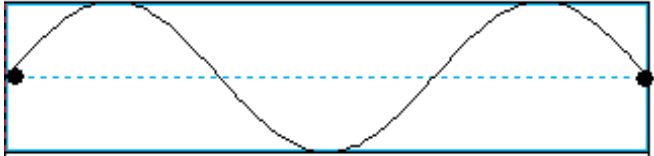
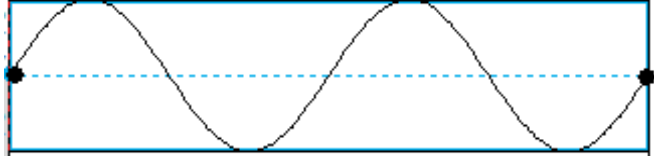
$L$  fits half of the wave

$L$  is half as long as  $\lambda_1$

$$\lambda_1 = 2L$$

## 2. Calculating resonance wavelengths

- **Node/node** (half-wavelength) system  
Tube or string of length  $L$

	$L = \frac{1}{2} \cdot \lambda_1$	$\lambda_1 = 2L$	$\lambda_1 = (2/1) \cdot L$
	$L = 1 \cdot \lambda_2$	$\lambda_2 = L$	$\lambda_2 = (2/2) \cdot L$
	$L = 1\frac{1}{2} \cdot \lambda_3$	$\lambda_3 = \frac{2}{3}L$	$\lambda_3 = (2/3) \cdot L$
	$L = 2 \cdot \lambda_4$	$\lambda_4 = \frac{1}{2}L$	$\lambda_4 = (2/4) \cdot L$

- **General formula:**

## 2. Calculating resonance wavelengths

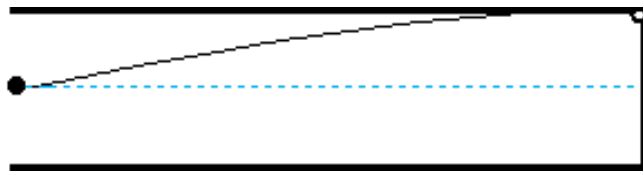
- **Node/node** (half-wavelength) system  
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	$L = 2 \cdot \lambda_4$	$\lambda_4 = \frac{1}{2}L$	$\lambda_4 = (2/4) \cdot L$

- **General formula:**  $\lambda_n = (2/n) L$  or  $\lambda_n = 2L/n$

## 2. Calculating resonance wavelengths

- Consider the **first resonance** of a **node/antinode** (quarter-wavelength) system



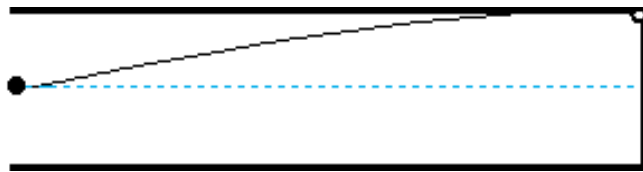
→  $\frac{1}{4}$  cycle of the wave "fits"

- If the tube is length  $L$ , what is the **wavelength** of the first resonance ( $\lambda_1$ )?



## 2. Calculating resonance wavelengths

- Consider the **first resonance** of a **node/antinode** (quarter-wavelength) system



→  $\frac{1}{4}$  cycle of the wave "fits"

- If the tube is length  $L$ , what is the **wavelength** of the first resonance ( $\lambda_1$ )?

$L$  fits one quarter of the wave

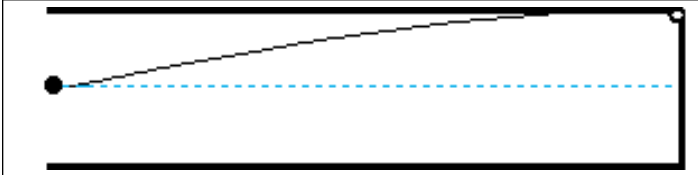
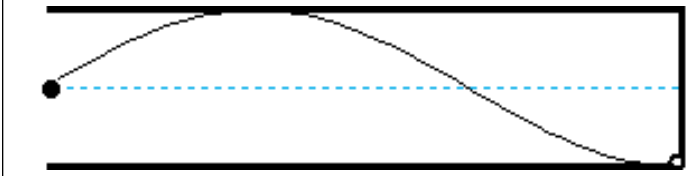
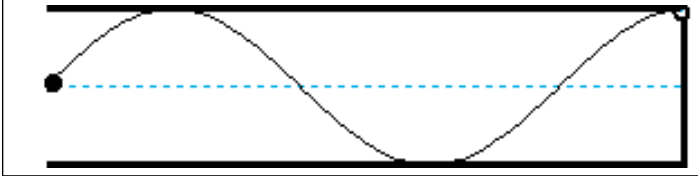
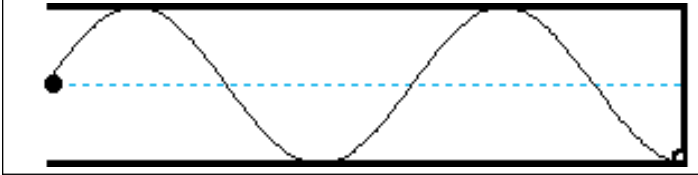
$L$  is one quarter as long as  $\lambda_1$

$$\lambda_1 = 4L$$

## 2. Calculating resonance wavelengths

- **Node/antinode** (quarter-wavelength) system

Tube of length  $L$

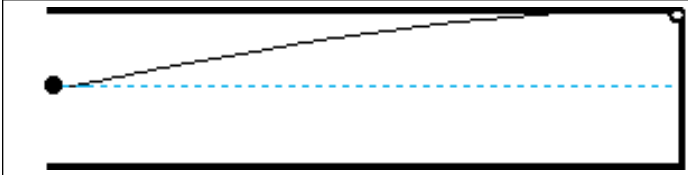
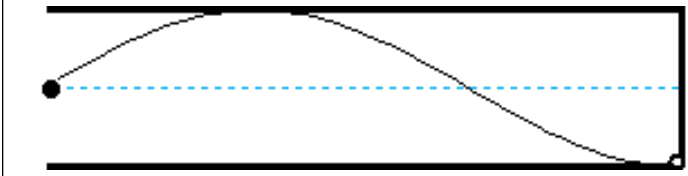
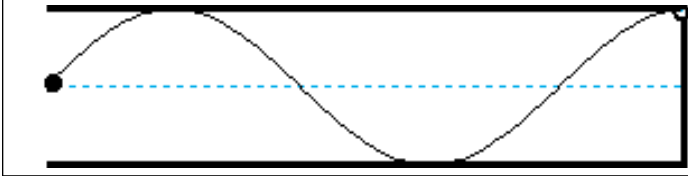
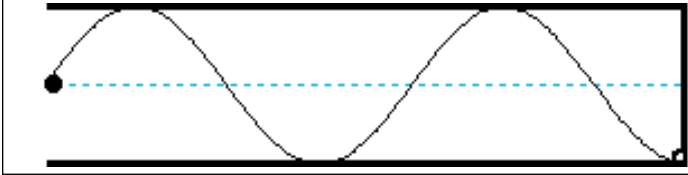
	$L = \frac{1}{4} \cdot \lambda_1$	$\lambda_1 = 4 L$	$\lambda_1 = (4/1) \cdot L$
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	$L = 1\frac{1}{4} \cdot \lambda_3$	$\lambda_3 = \frac{4}{5} L$	$\lambda_3 = (4/5) \cdot L$
	$L = 1\frac{3}{4} \cdot \lambda_4$	$\lambda_4 = \frac{4}{7} L$	$\lambda_4 = (4/7) \cdot L$

- **General formula:**

## 2. Calculating resonance wavelengths

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Tube of length  $L$

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- **General formula:**  $\lambda_n = (4/(2n-1)) \cdot L$  or  $\lambda_n = 4L / (2n-1)$

### 3. Calculating resonance frequencies

- Finally!—the **frequencies** of the resonances are what we really want to know
  - Knowing the resonance frequencies of a tube of air (in the vocal tract) helps us **model the acoustics of speech sounds**
- *Example:* Soon we will learn about the **source-filter model** of speech acoustics as applied to **vowels**
  - The **source** is the glottal-source wave
  - The **filter** is determined by the **resonance frequencies of the vocal tract**

### 3. Calculating resonance frequencies

- **Wavelength ( $\lambda$ )** and **frequency ( $f$ )** are related:

$$c = \lambda f$$

- where  **$c$**  is the speed of the wave (about **350 m/s** for sound in air, according to *AAP*)
- Wavelength and frequency are ***inversely*** related
  - **Long** wavelength means **low** frequency
  - **Short** wavelength means **high** frequency

Imagine traffic moving by at a steady 35 mph. Many VW bugs (short) would go by in 1 minute (higher frequency), but few buses (long) would go by in 1 minute (lower frequency).

### 3. Calculating resonance frequencies

- If we know wavelength, we can **solve for frequency**

$$c = \lambda f$$

$$f = c / \lambda$$

- Find the **frequency** of the ***n*th resonance ( $f_n$ )**:
  - Reminder: What is  $c$ ?
  - Plug the wavelength  $\lambda_n$  into the formula
  - Solve for  $f_n$

### 3. Calculating resonance frequencies

- For a **node/node** system with tube of length  $L$

$$\lambda_n = 2L/n$$

| relates wavelength to tube length

$$f_n = c/\lambda_n$$

| relates frequency to wavelength

$$f_n = c / (2L/n)$$

| relates frequency to tube length

$$f_n = n \cdot c/2L$$

- **Shortcut!** Once you know the **1st resonance  $f_1$**  :

$$f_n = \underline{\hspace{2cm}}$$

### 3. Calculating resonance frequencies

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- **Shortcut!** Once you know the **1st resonance  $f_1$**  :

$$f_n = n \cdot f_1 \quad | \quad \text{because } f_1 = 1 \cdot c / 2L = c / 2L$$

→ The resonance frequencies in a **node/node** system are **whole-number multiples** of  $f_1$



### 3. Calculating resonance frequencies

- For a **node/antinode** system with tube of length  $L$

$$\lambda_n = 4L / (2n-1)$$

| relates wavelength to tube length

$$f_n = c / \lambda_n$$

| relates frequency to wavelength

$$f_n = c / (4L / (2n-1))$$

| relates frequency to tube length

$$f_n = (2n-1) \cdot c / 4L$$

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### 3. Calculating resonance frequencies

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- **Shortcut!** Once you know the **1st resonance  $f_1$**  :

$$\mathbf{f_n = (2n-1) \cdot f_1} \quad | \quad \text{because } \mathbf{f_1 = 1 \cdot c / 4L = c / 4L}$$

→ The resonance frequencies in a **node/antinode** system are **odd-numbered multiples** of  $f_1$

### 3. Calculating resonance frequencies

- What is the **relationship** between  $f_1$  (the **first resonance** frequency) and  $f_0$  (the **fundamental frequency** of the complex wave itself) for...
  - a **node/node** system?
  - a **node/antinode** system?

### 3. Calculating resonance frequencies

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    - Resonance  $f$ s = whole-number multiples of  $f_1$   
What does this tell us?
  - a **node/antinode** system?
    - Resonance  $f$ s = odd-numbered multiples of  $f_1$   
What does this tell us?

## 4. The glottal source wave

- What is the **glottal source wave**?
  - Also called the **voicing wave**(form) in *AAP* Ch 2
  - The sound wave produced by \_\_\_\_\_

## 4. The glottal source wave

- What is the **glottal source wave**?
  - Also called the **voicing wave**(form) in *AAP* Ch 2
    - The sound wave produced by the vibration of the vocal folds
- To actually hear this sound wave, you would have to put a microphone right above the glottis
  - The sound waves of any speech we normally hear are **further modified** by passing through the vocal tract
    - This is the content of the rest of the course!

## 4. The glottal source wave

- The glottal source wave is a complex wave with the following property:
  - All of the components of this complex wave have frequencies that are **whole-number multiples** of the lowest-component frequency
- How does the fundamental frequency of the glottal source wave **relate** to the frequency of its lowest component?