# Modeling vowel formants —

- **Multiple-tube model**
- **Perturbation theory**

*Background reading:*

- *AAP* sec 6.1, “Tube models of vowel production”

# 0. Today's plan

- The multiple-tube model of vowel formants
  - From vowel constrictions to a series of tubes
  - Resonance frequencies for tubes
  - Helmholtz resonator
  - Predict formants for [ɑ] *(prep questions)*, [i]
- Perturbation theory and modeling vowel formants
  - Vowel constrictions as tube perturbations
  - Nodes and antinodes in the vocal-tract tube
  - Perturbation "rules of thumb"
  - *Try at home:* Predict formants for [i ɑ u]

# 1. Review: Vowels as vocal-tract tubes

[ɑ]

[i]

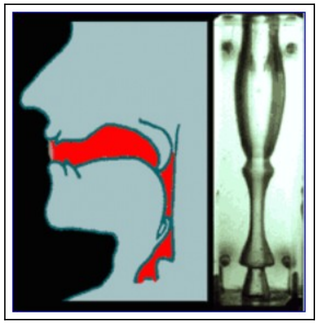
[u]

- How are these vowels described (in terms of height, etc.)?

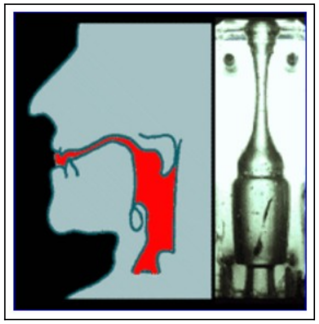
# 1. Review: Vowels as vocal-tract tubes

(images from Exploratorium)

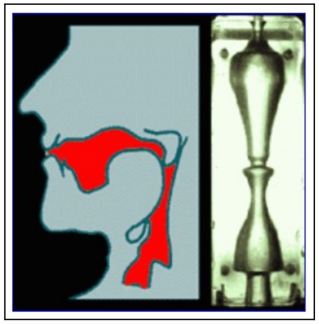
[ɑ] low back unrounded



[i] high front unrounded



[u] high back round



- Where does each vowel have a **constriction** (narrowing)? How is each **vocal-tract shape** as a series of **tubes**?

# 1. Review: Vowels as vocal-tract tubes

[ɑ]
- Narrowing in the **pharynx** (vertical part of vocal tract downstream of velum/uvula)
- **Wide tube** in front, **narrow tube** in back

[i]
- Narrowing at the **palate**
- **Wide tube** in front, **small narrow tube** in middle, **wide tube** in back

[u]
- Narrowing at the **velum** and **lips**
- (**Longer) wide tube** in front, **small narrow tube** in middle, **wide tube** in back + **lip rounding**

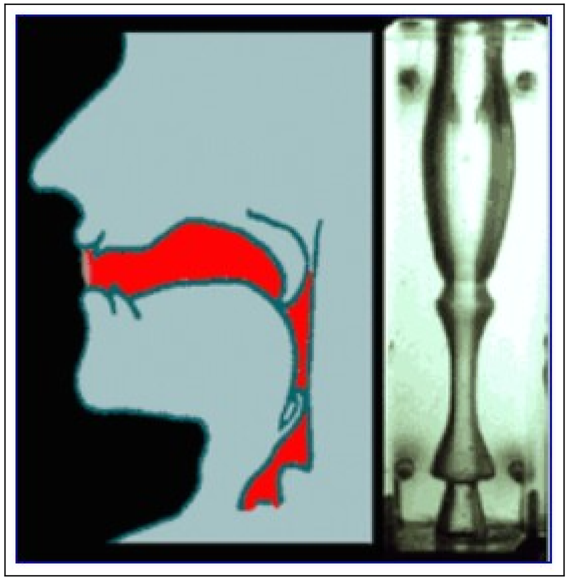
- We can use our understanding of **resonance frequencies in tubes** to model **vowel formants**

# 2. The mulitiple-tube model: Formants of [ɑ]

- Note:  Central [a] is more typical in a 5-vowel system, while languages with more vowels (such as English) may use back [ɑ]
- The acoustics (and articulations) of these vowels are similar; we will follow *AAP* and discuss [ɑ] in the multiple-tube model

# 2. The mulitiple-tube model: Formants of [ɑ]

- [ɑ] has a narrowing in the **pharynx**
- We can model this as a series of ***two*** tubes
  - A **wide** tube in front (top of pharynx to lips)
  - A **narrow** tube in back (glottis to top of pharynx)



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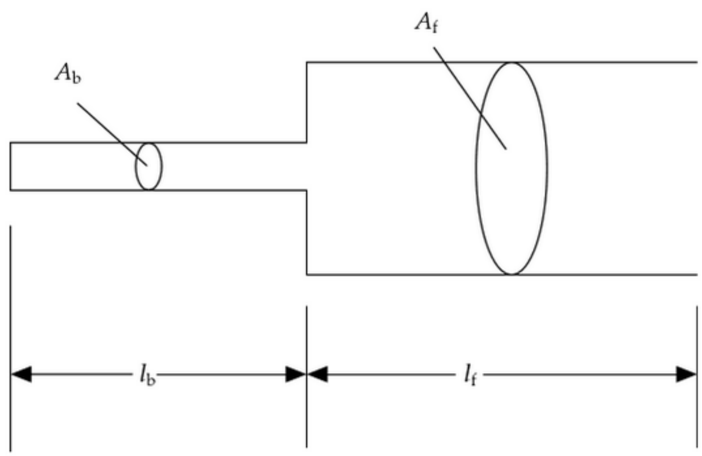



**Figure 6.1** Two-tube model of the vocal tract that approximates the shape of the vocal tract for [ɑ].

*AAP*, Fig 6.1 (note lips on *right* side)

# 2. The mulitiple-tube model: Formants of [ɑ]

- We can model [ɑ] as a series of two tubes
  - A **wide** tube in front (top of pharynx to lips)
  - A **narrow** tube in back (glottis to top of pharynx)
- What are the **boundary conditions** for these tubes?

# 2. The mulitiple-tube model: Formants of [ɑ]

- What are the **boundary conditions** for these tubes?

  **Front**-cavity tube:

  - *open* at the lips
  - *closed* at the other end (as far as wave reflection is concerned — back-cavity tube has a smaller diameter)

  **Back**-cavity tube:

  - *closed* at the glottis
  - *open* at the other end (as far as wave reflection is concerned — front-cavity tube has *larger* diameter)

- **Nodes** or **antinodes**?

# 2. The mulitiple-tube model: Formants of [ɑ]

- What are the **boundary conditions** for these tubes?

  **Front**-cavity tube:

  - *open* at the lips
  - *closed* at the other end (as far as wave reflection is concerned — back-cavity tube has a smaller diameter)

  **Back**-cavity tube:

  - *closed* at the glottis
  - *open* at the other end (as far as wave reflection is concerned — front-cavity tube has *larger* diameter)

- Each tube → **node/antinode** (quarter-wave) system!

# 2. The mulitiple-tube model: Formants of [ɑ]

- **Front**-cavity tube is a node/antinode system
  **Back**-cavity tube is a node/antinode system
  → What do we need to know in order to calculate the **resonance frequencies** of these tubes?

# 2. The mulitiple-tube model: Formants of [ɑ]

- **Front**-cavity tube is a node/antinode system
  **Back**-cavity tube is a node/antinode system
  → What do we need to know in order to calculate the **resonance frequencies** of these tubes?
  - All we need is **tube length** and $f_n = (2n\text{–}1) \bullet c/4L$

- **Try it:** Assume a speaker with vocal tract = 16cm, back cavity = 7cm (and therefore front cavity = 9cm)

# 2. The mulitiple-tube model: Formants of [ɑ]

- **Front**-cavity tube is a node/antinode system
  **Back**-cavity tube is a node/antinode system
  - All we need is **tube length** and $\boldsymbol{f_n = (2n\text{–}1) \bullet c/4L}$

- **Try it:** Assume a speaker with vocal tract = 16cm, back cavity = 7cm (and therefore front cavity = 9cm)
  - **Front**-cavity resonances (at $c$=350m/s)
    $f_{f1}$ = **972Hz**, $f_{f2}$ = 3*972Hz = **2916Hz**, ...
  - **Back**-cavity resonances (at $c$=350m/s)
    $f_{b1}$ = **1250Hz**, $f_{b2}$ = 3*1250Hz = **3750Hz**, ...

# 2. The mulitiple-tube model: Formants of [ɑ]

- What are the **formant frequencies of this speaker's [ɑ]** as predicted by this model?

## 2. The mulitiple-tube model: Formants of [ɑ]

- What are the **formant frequencies of this speaker's [ɑ]** as predicted by this model?
  - Remember: Formants are the **resonances of the vocal tract** when a vowel is produced

- So...what are the resonance frequencies produced by this two-tube system **as a whole**?
  - **Front**-cavity resonances (at $c$=350m/s)
    $f_{f1}$ = **972Hz**, $f_{f2}$ = 3*972Hz = **2916Hz**, ...
  - **Back**-cavity resonances (at $c$=350m/s)
    $f_{b1}$ = **1250Hz**, $f_{b2}$ = 3*1250Hz = **3750Hz**, ...

# 2. The mulitiple-tube model: Formants of [ɑ]

- What are the **formant frequencies of this speaker's [ɑ]** as predicted by this model?
  - Remember: Formants are the **resonances of the vocal tract** when a vowel is produced
- So…what are the resonance frequencies produced by this two-tube system **as a whole**?

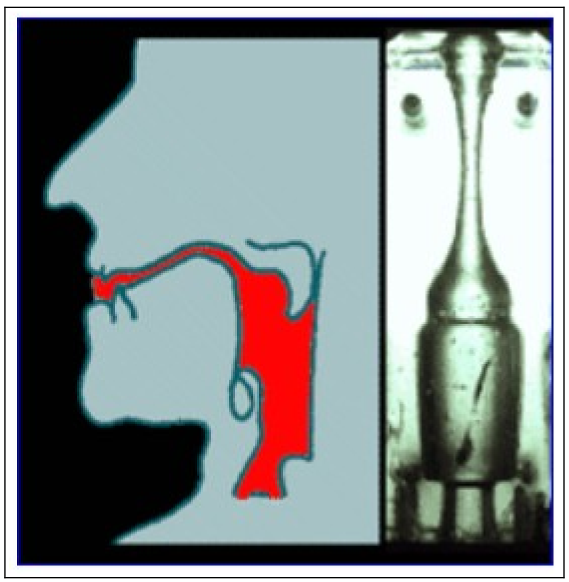
→ The front and back tube resonances all contribute! (List them in increasing order of $f$)

F1 = **972Hz** ($f_{f1}$)  F3 = **2916Hz** ($f_{f2}$)

F2 = **1250Hz** ($f_{b1}$)  F4 = **3750Hz** ($f_{b2}$)

# 3. The mulitiple-tube model: Formants of [i]

- [i] has a narrowing at the **palate**
- We can model this as a series of ***three*** tubes
  - A **wide** tube in front (palate to lips)
  - A **small narrow** tube at the palate
  - A **wide** tube in back (glottis to palate)



from [Exploratorium]

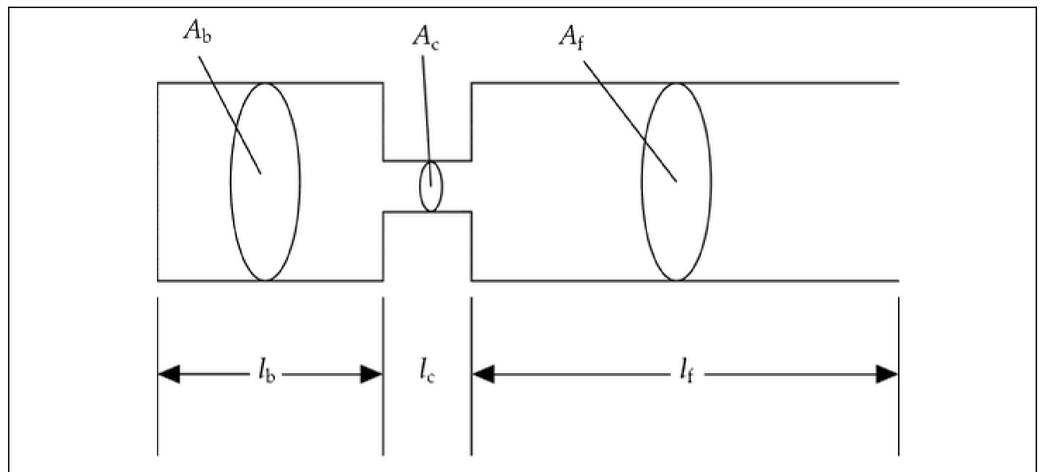



**Figure 6.3** Tube model of vocal tract configurations that have a short constriction at some point in the vocal tract.

*AAP*, Fig 6.3 (note lips on *right* side)

# 3. The mulitiple-tube model: Formants of [i]

- We can model [i] as a series of three tubes
  - A **wide** tube in front (palate to lips)
  - *A **small narrow** tube at the palate*
  - A **wide** tube in back (glottis to palate)
- What are the **boundary conditions** for the **wide** tubes?
- *The small narrow tube is modeled as a Helmholtz resonator; we will return to this shortly*

# 3. The mulitiple-tube model: Formants of [i]

- What are the **boundary conditions** for these tubes?

  **Front**-cavity tube:

  - *open* at the lips
  - *closed* at the other end (as far as wave reflection is concerned — narrow center tube has a *smaller* diameter)

  **Back**-cavity tube:

  - *closed* at the glottis
  - *closed* at the other end (as far as wave reflection is concerned — narrow center tube has a *smaller* diameter)

- **Nodes** or **antinodes**?

# 3. The mulitiple-tube model: Formants of [i]

- What are the **boundary conditions** for these tubes?

**Front**-cavity tube:

- *open* at the lips
- *closed* at the other end (as far as wave reflection is concerned — narrow center tube has a *smaller* diameter)

→ **Node/antinode** (***quarter***-wave) system

**Back**-cavity tube:

- *closed* at the glottis
- *closed* at the other end (as far as wave reflection is concerned — narrow center tube has a *smaller* diameter)

→ **Antinode/antinode** (***half***-wave) system

# 3. The mulitiple-tube model: Formants of [i]

- **Front**-cavity tube is a node/antinode system
  **Back**-cavity tube is an antinode/antinode system
  → What do we need to know in order to calculate the **resonance frequencies** of these tubes?

# 3. The mulitiple-tube model: Formants of [i]

- **Front**-cavity tube is a node/antinode system
  **Back**-cavity tube is an antinode/antinode system
  → What do we need to know in order to calculate the **resonance frequencies** of these tubes?
  - We need **tube length** and the two formulas
    $\boldsymbol{f_n = (2n–1) \cdot c/4L}$ and $\boldsymbol{f_n = n \cdot c/2L}$
- **Try it:** Assume a speaker with vocal tract = 16cm, back cavity = 10cm, narrow central constriction = 3cm (and therefore front cavity = 3cm)

# 3. The mulitiple-tube model: Formants of [i]

- **Front**-cavity tube is a node/antinode system
  **Back**-cavity tube is an antinode/antinode system
  - We need **tube length** and the two formulas
    $\boldsymbol{f_n = (2n\text{–}1) \bullet c/4L}$ and $\boldsymbol{f_n = n \bullet c/2L}$
- **Try it:** Assume a speaker with vocal tract = 16cm, back cavity = 10cm, constriction = 3cm, front = 3cm
  - **Front**-cavity resonances (at *c*=350m/s)
    $f_{f1}$ = **2917Hz** , $f_{f2}$ = 3*2917Hz = **8750Hz**, ...
  - **Back**-cavity resonances (at *c*=350m/s)
    $f_{b1}$ = **1750Hz** , $f_{b2}$ = **2***1750Hz = **3500Hz**, ...

# 3. The mulitiple-tube model: Formants of [i]

- Now we can look at the narrow central constriction (at the palate for [i])
  - This constriction also has its own resonance frequencies, as a (very short!) tube, but they are too high to matter for speech analysis
- This constriction plus the back cavity form a **Helmholtz resonator**, which also has a (single!), low-frequency resonance
  - A Helmholtz resonator involves a small volume of air that oscillates into and out of a larger one
  - For an animation, see "Helmholtz Resonance" (UNSW); scroll down to the *bottle* animation

# 3. The mulitiple-tube model: Formants of [i]

- There is a formula for calculating the Helmholtz resonance in this three-tube vowel system, based on the relative volumes of the central constriction and the back tube (see *AAP*, p 135, equation 6.2)
  - You do not need to know this formula
  - We will just **estimate** the Helmholtz resonance in a high vowel as being around **200Hz**

# 3. The mulitiple-tube model: Formants of [i]

- Putting it all together:
  - **Front**-cavity resonances (at *c*=350m/s)

    $f_{f1}$ = **2917Hz**, $f_{f2}$ = 3*2917Hz = **8750Hz**, ...

  - **Back**-cavity resonances (at *c*=350m/s)

    $f_{b1}$ = **1750Hz** , $f_{b2}$ = **2***1750Hz = **3500Hz**, ...

  - Helmholtz resonance = around **200Hz**

# 3. The mulitiple-tube model: Formants of [i]

- What are the **formant frequencies of this speaker's [i]** as predicted by this model?

# 3. The mulitiple-tube model: Formants of [i]

- What are the **formant frequencies of this speaker's [i]** as predicted by this model?

- List them all in order

  F1 = **200Hz** (Helmholtz resonance)

  F2 = **1750Hz** ($f_{b1}$) | note—a little low for a typical [i]

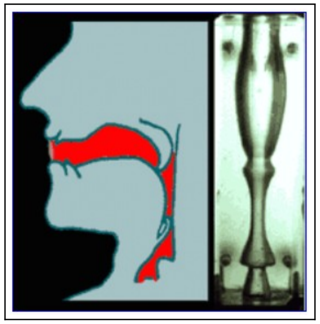
  F3 = **2917Hz** ($f_{f1}$)

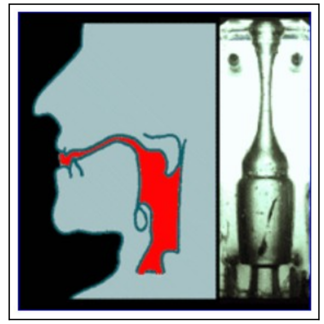
  …

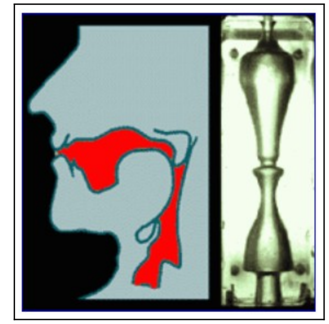
# 4. The mulitiple-tube model: Some final points

- Any vowel can in principle be modeled with this multiple-tubes model
  - It might need more than three tubes…
- For this class, we will focus on understanding the [ɑ]-type vowels and [i]-type vowels discussed above
- A **nomogram** is a graph that displays resonances as calculated for tube systems of particular sizes
  - See AAP, sec 6.1, reproduced on this handout
  - Check out the nomograms and try to relate them to today's discussion

# 5. Perturbation theory: Overview

[ɑ] 

[i] 

[u] 

low back unrd | high front unrd | high back round

- Using the **multiple-tube model**, we can model vowel **vocal-tract shapes** as a ***series*** of **tubes**
- With **perturbation theory**, we can model vowel **vocal-tract shapes** as a **perturbation** (modification) of a ***uniform*** **tube**
- Both models are simplifications, but are useful ways of understanding and predicting speech acoustics

(images from Exploratorium)

# 5. Perturbation theory: Overview

- We can use our understanding of vowel articulations as **narrowings** in the vocal tract…
  - to model expected **deviations** in the **resonance frequencies** from those of a **uniform tube** ([ə])
  - and thereby **predict formants** of non-[ə] vowels

- Later in the course, we will also use perturbation theory to model **place-of-articulation** effects in consonant acoustics

# 5. Perturbation theory: Overview

- First step:  Model the formant frequencies of [ə] (uniform vocal-tract tube)
- Then:  Predict how formant frequencies will differ in other speech sounds
  - Find **where** there is a narrow region in the vocal tract—where the uniform tube is "perturbed"
  - Determine how a perturbation at that vocal-tract location should **change** the resonance frequencies of the tube
    - Note:  Consider each resonance separately

# 6. Review: Vowel articulations for [i ɑ u]

- **Fill in the chart** on the next slide
  *(download and save the PDF if you want to keep your work)*
  - Complete the articulatory descriptions
  - State whether or not each vowel has a narrowing at the indicated point in the vocal tract

- Reminders
  - Narrowing at the *lips* when a vowel is *round*
  - Narrowing at the *palate* when a vowel is *high* and *front*
  - Narrowing at the *velum* when a vowel is *high* and *back*
  - Narrowing at the *pharynx* when a vowel is *low* (especially if also *back*)

# 6. Review: Vowel articulations for [i ɑ u]

- Fill in the chart: Where is there a constriction?

| vowel | **description** | **lips?** | **palate?** | **velum?** | **pharynx?** |
|---|---|---|---|---|---|
| **[i]** | | ☐ | ☐ | ☐ | ☐ |
| **[ɑ]** | | ☐ | ☐ | ☐ | ☐ |
| **[u]** | | ☐ | ☐ | ☐ | ☐ |

"Description" = high/mid/low, front/central/back, round/unrounded

# 7. Review: Nodes & antinodes in v. t. tube

- Model the vocal tract as a uniform tube that is **open** at the lips and **closed** at the glottis
- Consider the **pressure** wave:
  - What are the **boundary conditions**?
    Node?  Antinode?
  - For the **first three resonances**, where are *all* of the nodes and antinodes in the tube?
- Review: "Standing Sound Waves" animation of standing waves in air in a tube (Dan Russell, Penn State)
  - Graphs compare the pressure and displacement waves, and relate both to the air molecules in the tube

# 7. Review: Nodes & antinodes in v. t. tube

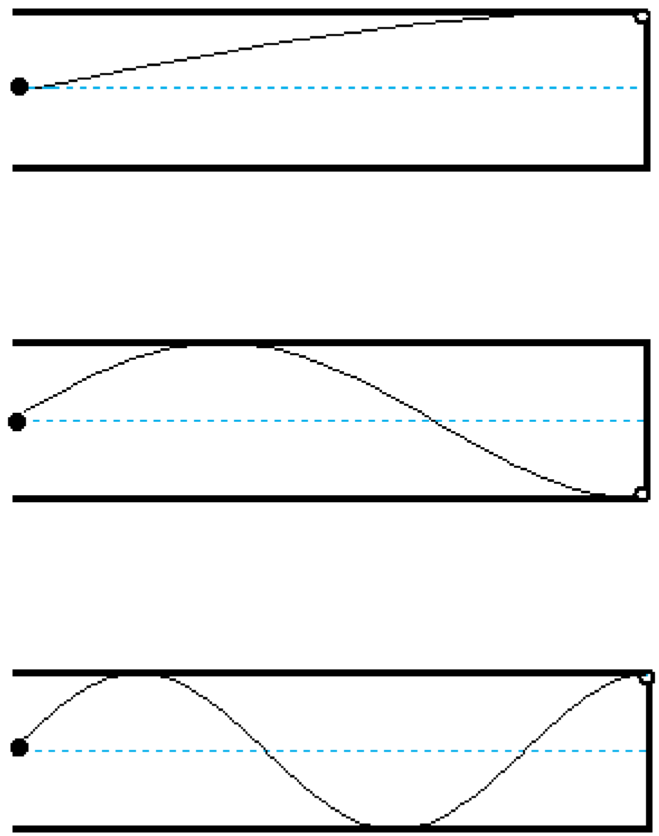
- What we saw before:  The first three resonances
  Where are *all* the **nodes** and **antinodes** for each?

*(lips)* *(glottis)*

*1st resonance*

*2nd resonance*

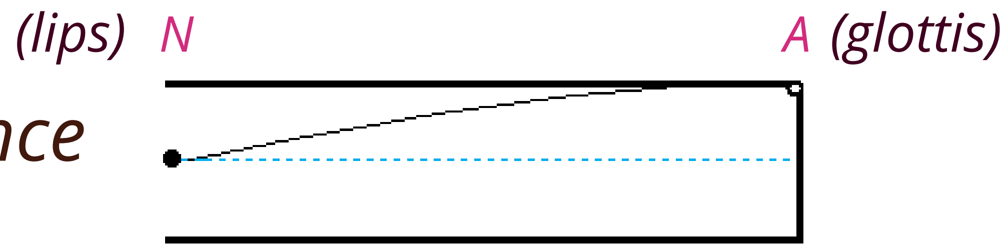
*3rd resonance*



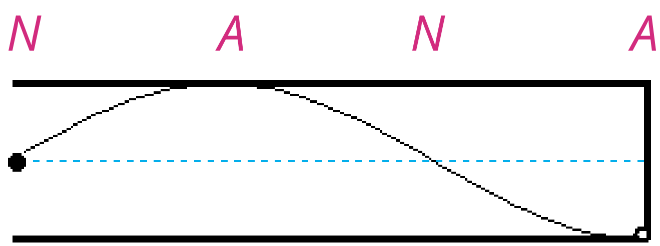
# 7. Review: Nodes & antinodes in v. t. tube

- What we saw before: The first three resonances
  Where are *all* the **nodes** and **antinodes** for each?

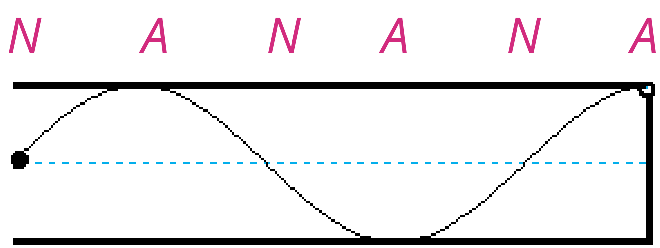
*1st resonance*




*2nd resonance*




*3rd resonance*



# 7. Review: Nodes & antinodes in v. t. tube

- Compare the diagrams in *AAP* Fig. 6.7 (p 139): These show the first four resonances of an open/closed tube, superimposed on the vocal tract

Some points to consider here:

- These diagrams look “backward” because they show the velocity wave (or displacement wave), which has *nodes* where the pressure waves we draw have *antinodes*
- The points labeled V, V’, V”, etc. show antinodes on the **v**elocity waves, and so *nodes* on the pressure waves

- Again, see “Standing Sound Waves” for pressure vs. displacement(=velocity) standing wave diagrams

# 8. Perturbing the tube

- Perturbation theory uses two “rules of thumb” to predict what will happen to a resonance frequency when a tube has a narrowing at a node or antinode
- Use the discussion in *AAP* sec 6.2 to fill in the blanks on the following slide with ***up*** **or** ***down***
  - Remember that the *AAP* “node”/”antinode” descriptions are given in terms of the **velocity** or displacement wave, not the **pressure** wave!

# 8. Perturbing the tube

- **Perturbation rules — MEMORIZE THIS**

1. If there is a narrowing in the vocal tract near a velocity/displacement antinode = **pressure node**, the **formant frequency goes** ______.

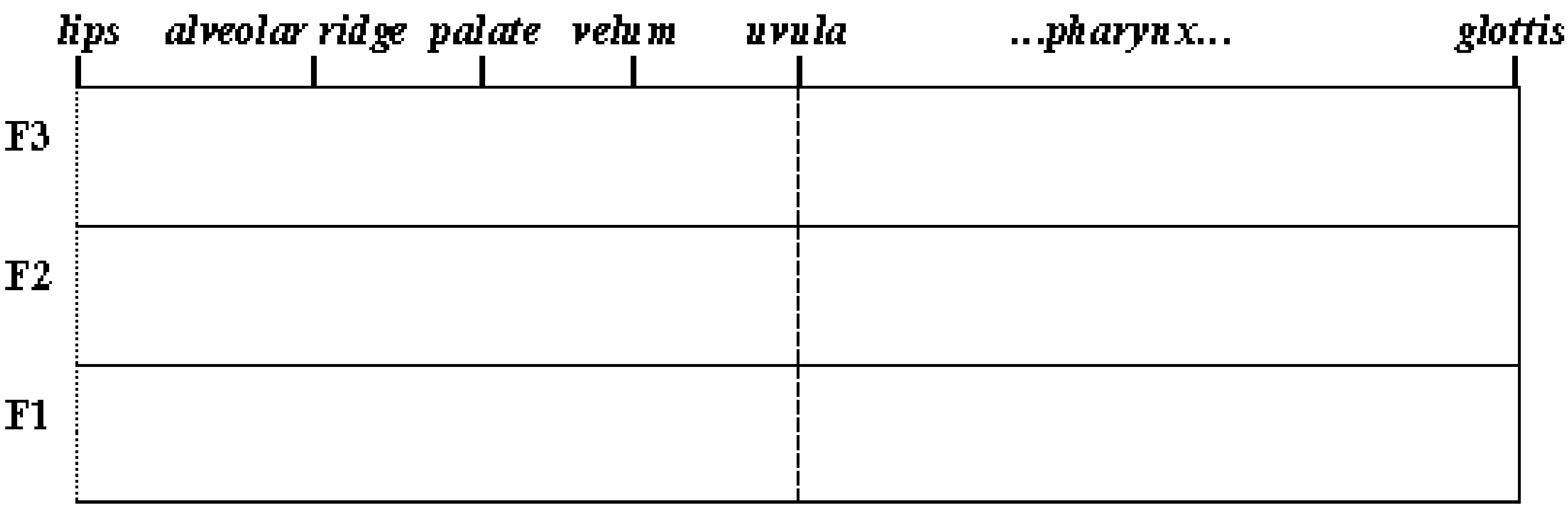
2. If there is a narrowing in the vocal tract near a **pressure antinode** (velocity/displacement node), **formant frequency goes** ______.

# 8. Perturbing the tube

- Forming a constriction or narrowing in the vocal tract **affects each formant separately**
  - The effect on each formant depends on whether the narrowing is closer to a (pressure) node or closer to a (pressure) antinode **for that formant**
  - The **same point** in the vocal tract could be near a node for one formant and an antinode for another!

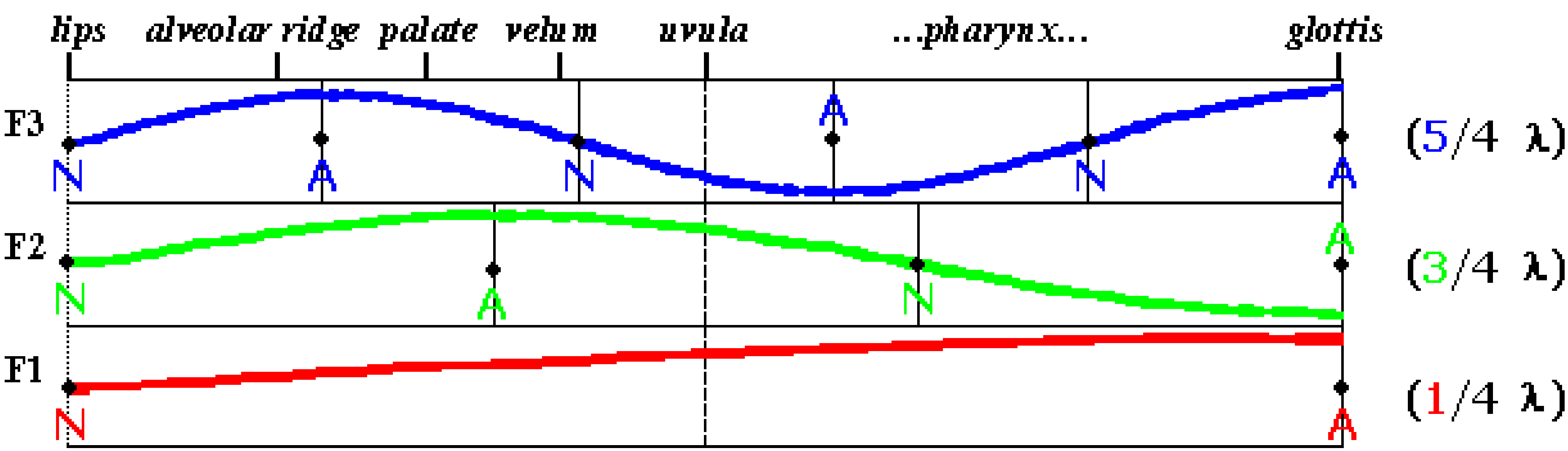
# 8. Perturbing the tube

- Articulatory landmarks in the vocal tract
  - The following diagram shows approximately where different vocal-tract structures lie along the length of the vocal tract



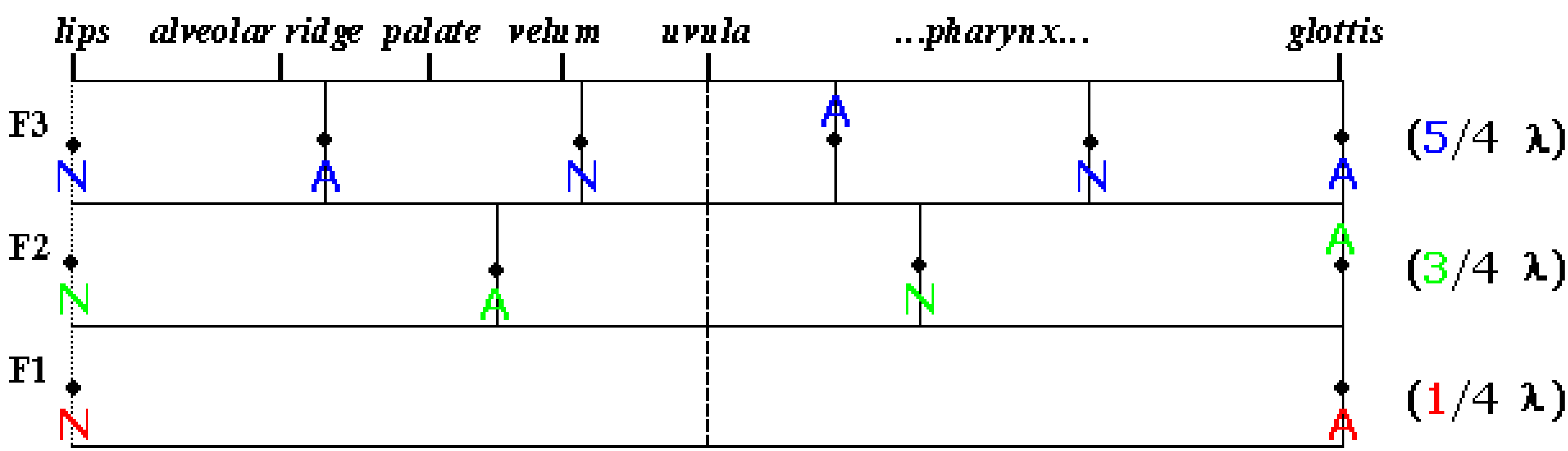
# 8. Perturbing the tube

- Articulatory landmarks in the vocal tract
  - Draw the first three standing (pressure) waves
  - Label their **nodes** and **antinodes**



# 8. Perturbing the tube

- Articulatory landmarks in the vocal tract
  - What we need to pay attention to is not the standing-wave diagram itself, but specifically where the **nodes** and **antinodes** are



# 8. Perturbing the tube

- This tube diagram of the vocal tract is based on *AAP* Fig 6.12 (p 151), for the "typical male speaker". Landmarks:
  - uvula at midpoint of vocal tract
  - alveolar ridge at 1/3 of the distance from lips to uvula
  - palate at 1/3 and velum at 2/3 of the distance from the alveolar ridge to the uvula

# 9. Predicting formants in non-[ə] vowels

- Now you have what you need to use perturbation theory to make predictions about vowel formants!
- Is a particular narrowing in the vocal tract *closer* to a (pressure) node, or *closer* to a (pressure) antinode?
  → This predicts what the formants will do

# 9. Predicting formants in non-[ə] vowels

**Try it out — we'll check in next time**

- Use what you know about:
  - vowel articulations (slide 34)
  - the perturbation rules (slide 40)
  - locations of the (pressure) nodes/antinodes in the vocal tract (slide 44)

  to fill in the charts on the next two slides
  (except F2 for [u])

# 9. Predicting formants in non-[ə] vowels

- Should the formant be higher (↑) or lower (↓) than the equivalent formant in [ə] when there is a narrowing as indicated?

| | lips | palate | velum | pharynx |
|---|---|---|---|---|
| F3 | ○ ↑ ○ ↓ | ○ ↑ ○ ↓ | ○ ↑ ○ ↓ | ○ ↑ ○ ↓ |
| F2 | ○ ↑ ○ ↓ | ○ ↑ ○ ↓ | ○ ↑ ○ ↓ | ○ ↑ ○ ↓ |
| F1 | ○ ↑ ○ ↓ | ○ ↑ ○ ↓ | ○ ↑ ○ ↓ | ○ ↑ ○ ↓ |

# 9. Predicting formants in non-[ə] vowels

- Should the formants **in these vowels** be higher (↑) or lower (↓) than the equivalent formant in [ə]?

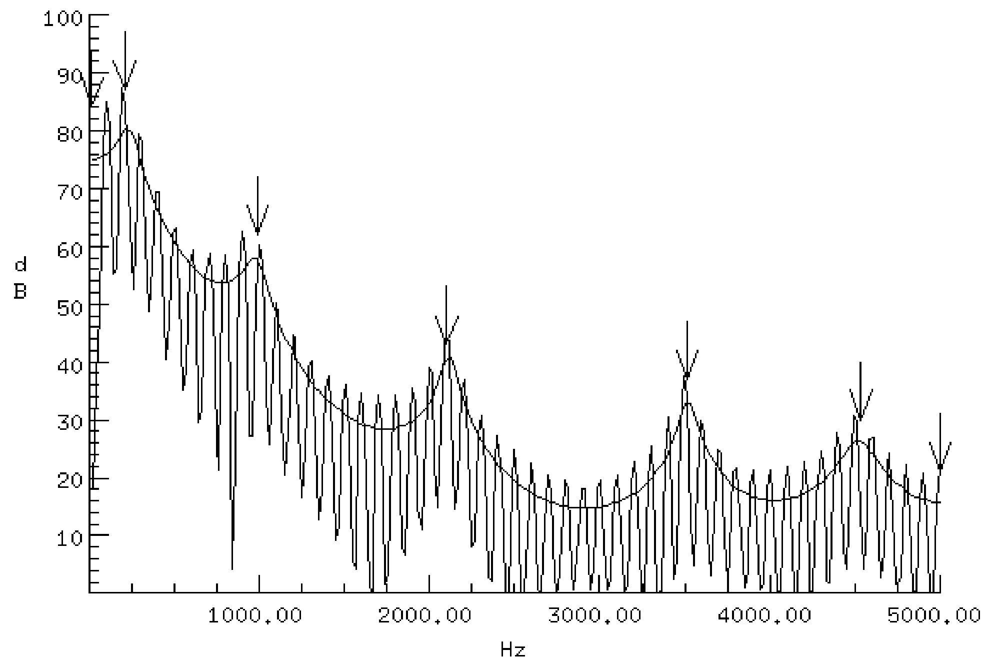
| | [i] | [ɑ] | [u] |
|---|---|---|---|
| F3 | ◯ ↑ ◯ ↓ | ◯ ↑ ◯ ↓ | ◯ ↑ ◯ ↓ |
| F2 | ◯ ↑ ◯ ↓ | ◯ ↑ ◯ ↓ | *see below* |
| F1 | ◯ ↑ ◯ ↓ | ◯ ↑ ◯ ↓ | ◯ ↑ ◯ ↓ |

# 9. Predicting formants in non-[ə] vowels

- Why is it hard to make a prediction for F2 in [u] using perturbation theory?

# 9. Predicting formants in non-[ə] vowels

- Look at this [u] spectrum; formants are indicated with arrows (from U Delaware Speech Research)
  - What does F2 in [u] actually look like compared to [ə]? ([ə] produced by this synthesizer: F2=1550 Hz)



# 9. Predicting formants in non-[ə] vowels

Extension to **mid vowels**: [e], [o]

- [e] is less high and less front than [i]
  - Its formant frequencies are perturbed in the direction of [i], but not as far
- Likewise, [o] is less high and less back than [u]
  - Its formant frequencies are perturbed in the direction of [u], but not as far
  - Note:  American English so-called “[u]” is more of a central vowel ([ʉ]) than a back one; [o] may be *further back* than [u]!