Thermodynamics of Gross-Neveu Models

Gökçe Başar

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06/29/2011

GB & G.Dunne arXiv:1011.3835 JHEP 01(2011)**127**
GB, G.Dunne & D. Kharzeev arXiv:1003.3464, PRL **104** 232301
GB, G.Dunne & M. Thies arXiv:0903.1868, PRD **79** 105012
GB & G.Dunne arXiv:0803.1501, PRL **100** 200404
arXiv:0806.2659, PRD **78** 065022

Outline

- Introduction to Gross-Neveu models
- Gap equation and inhomogeneous phases
- Phase diagrams
- Ginzburg-Landau expansion
- Connection to string theory

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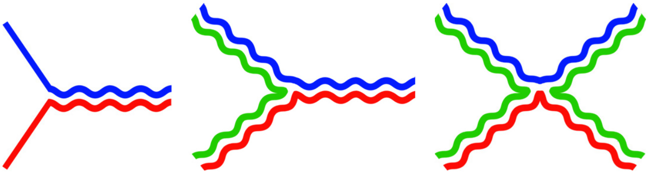
Quantum Chromodynamics (QCD)

Theory of quarks and gluons



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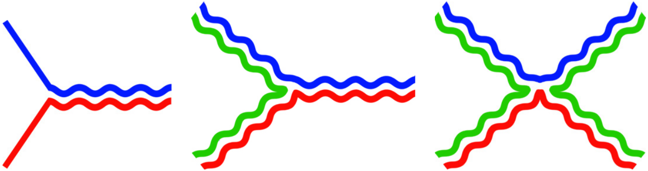
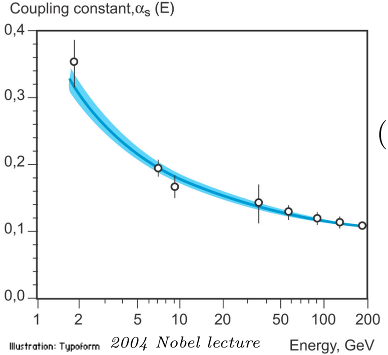
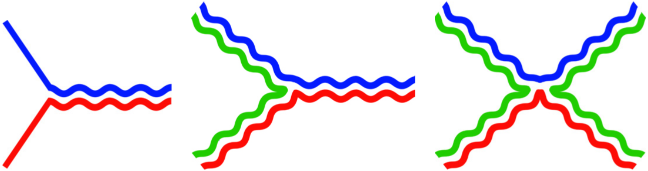
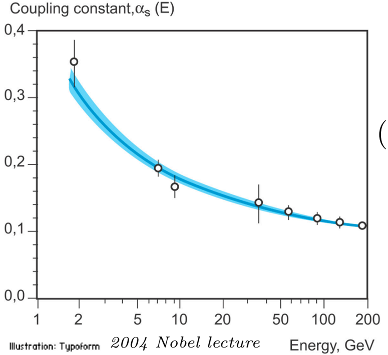




asymptotic freedom
(Gross, Wilczek, Politzer, 1973)

Theory of quarks and gluons



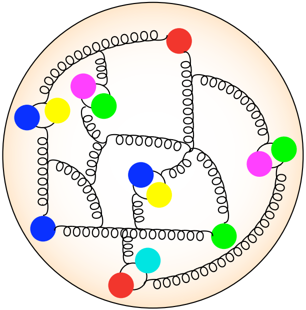


2004 Nobel lecture

"infrared slavery"
(Gross, Wilczek, Politzer, 1973)

Quantum Chromodynamics (QCD)

low energy excitations = quasi particles of strongly interacting quarks and gluons $(p, n, \pi, ...)$



- *heavy* ($m_q \sim$ MeV, $m_p \sim$ GeV)
- *color neutral*

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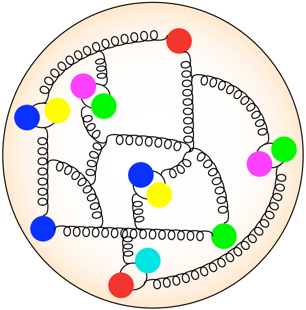


- *heavy* ($m_q \sim$ MeV, $m_p \sim$ GeV)

$\Updownarrow$

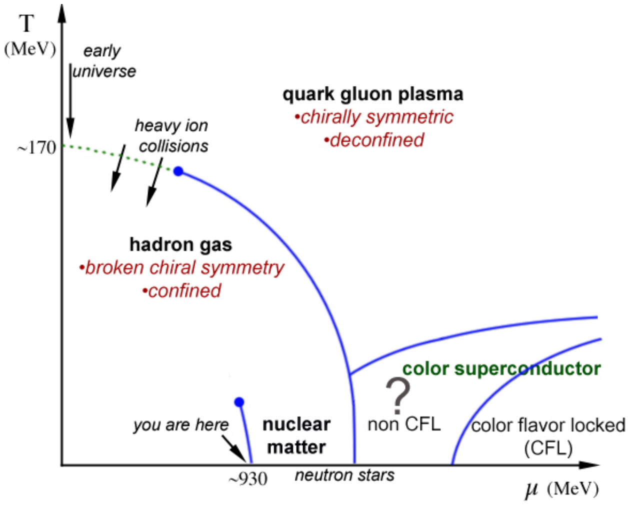
chiral symmetry breaking

- *color neutral*

$\Updownarrow$

confinement

Phase diagram of QCD?



Nambu-Jona-Lasinio Model

PHYSICAL REVIEW VOLUME 122, NUMBER 1 APRIL 1, 1961

Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity. I*

Y. NAMBU AND G. JONA-LASINIO†

The Enrico Fermi Institute for Nuclear Studies and the Department of Physics, The University of Chicago, Chicago, Illinois

(Received October 27, 1960)

It is suggested that the nucleon mass arises largely as a self-energy of some primary fermion field through the same mechanism as the appearance of energy gap in the theory of superconductivity. The idea can be put into a mathematical formulation utilizing a generalized Hartree-Fock approximation which regards real nucleons as quasi-particle excitations. We consider a simplified model of nonlinear four-fermion interaction which allows a γ_5-gauge group. An interesting consequence of the symmetry is that there arise automatically pseudoscalar zero-mass bound states of nucleon-antinucleon pair which may be regarded as an idealized pion. In addition, massive bound states of nucleon number zero and two are predicted in a simple approximation.

The theory contains two parameters which can be explicitly related to observed nucleon mass and the pion-nucleon coupling constant. Some paradoxical aspects of the theory in connection with the γ_5 transformation are discussed in detail.

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- Dynamical mass generation $\Rightarrow$ Chiral symmetry breaking

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- Dynamical mass generation ⇒ Chiral symmetry breaking
- Not renormalizable

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- Dynamical mass generation $\Rightarrow$ Chiral symmetry breaking
- Not renormalizable (in 3+1 dimensions)

Gross-Neveu Model

self interacting fermions in *two* spacetime dimensions

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self interacting fermions in *two* spacetime dimensions

$\Rightarrow$ Renormalizable

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self interacting fermions in *two* spacetime dimensions

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PHYSICAL REVIEW D VOLUME 10, NUMBER 10 15 NOVEMBER 1974

Dynamical symmetry breaking in asymptotically free field theories*

David J. Gross† and André Neveu
Institute for Advanced Study, Princeton, New Jersey 08540
(Received 21 March 1974)

- Asymptotically free!

self interacting fermions in *two* spacetime dimensions

⇒ Renormalizable

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Gross-Neveu and Nambu-Jona Lasinio Models

$$\mathcal{L}_{\text{GN}} = \bar{\psi}\, i\, \partial\!\!\!/\, \psi + \frac{g^2}{2} \left[\left(\bar{\psi}\psi \right)^2 \right]$$

(Gross, Neveu, 1974)

$$\mathcal{L}_{\text{NJL}} = \bar{\psi}\, i\, \partial\!\!\!/\, \psi + \frac{g^2}{2} \left[\left(\bar{\psi}\psi \right)^2 + \left(\bar{\psi} i \gamma^5 \psi \right)^2 \right]$$

(Nambu, Jona-Lasinio, 1961)

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discrete: $\psi \to \gamma^5 \psi$

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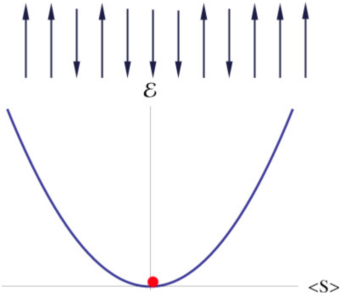
Spontaneous Symmetry Breaking



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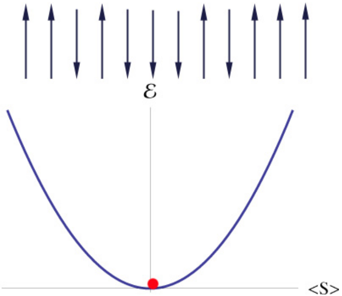

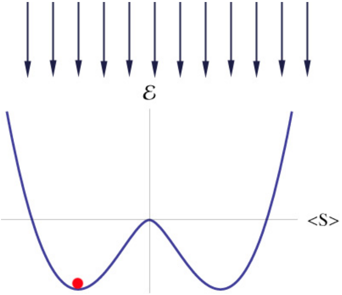

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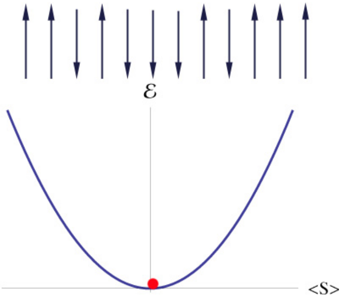
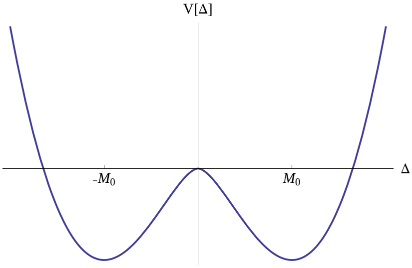


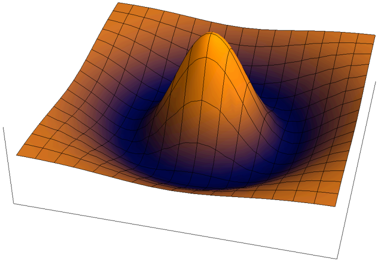
$\langle S \rangle \equiv \Delta$ order parameter

Spontaneous Symmetry Breaking

Gross-Neveu:



NJL:



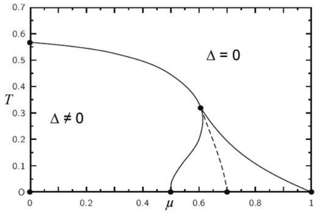
order parameter:

$$\langle\bar{\psi}\psi\rangle \sim \Delta$$

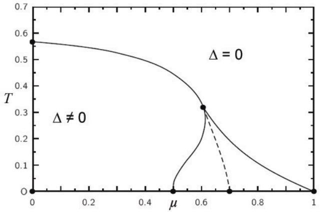
$$\langle\bar{\psi}\psi\rangle - i\langle\bar{\psi}i\gamma^5\psi\rangle \sim \Delta$$

$\Delta \neq 0$ breaks chiral symmetry

T-μ Phase Diagram ?



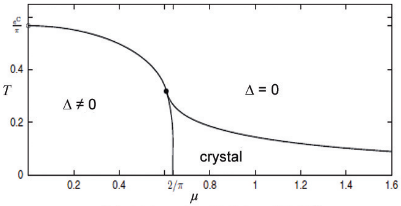
(U. Wolff, 1985)



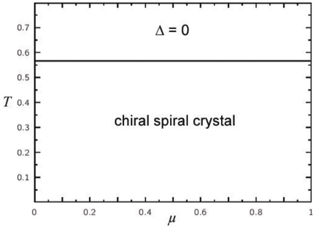
(A. Barducci et al., 1995)

GN_2

NJL_2



(M. Thies, K. Urlichs, 2005)
(P. de Forcrand, U. Wenger, 2006)



(GB, G. Dunne, M. Thies, 2009)

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- Translational symmetry breaking (at nonzero density)

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- Introduction to Gross-Neveu models
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- Phase diagrams
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Hubbard-Stratonovich transformation

Introduce a *complex* condensate : $\Delta(x) = S(x) - iP(x)$

where $\bar{\psi}\psi \to -S/g^2 \qquad \bar{\psi}i\gamma^5\psi \to -P/g^2$

$$\mathcal{L} = \bar{\psi}\left[i\,\partial\!\!\!/ - \frac{1}{2}(1-\gamma^5)\Delta - \frac{1}{2}(1+\gamma^5)\Delta^*\right]\psi - \frac{1}{2g^2}|\Delta|^2$$

$$H = -i\gamma^5\frac{d}{dx} + \gamma^0\left(\frac{1}{2}(1-\gamma^5)\Delta - \frac{1}{2}(1+\gamma^5)\Delta^*\right)$$

Hartree-Fock Approach

Dirac-Bogoliubov-de Gennes equation :

$$H\psi = E\psi$$

$$\text{H}=\begin{pmatrix} -i\frac{d}{dx} & \Delta(x) \\ \Delta^*(x) & i\frac{d}{dx} \end{pmatrix}$$

with consistency condition : $\langle\bar{\psi}\psi\rangle - i\langle\bar{\psi}i\gamma^5\psi\rangle = -\Delta/g^2$

"Inhomogeneous mean field approximation"

Gap Equation Approach

Effective action :

$$S_{\text{eff}}[\Delta] = -\frac{1}{2g^2 N_f} \int d^2x |\Delta|^2 - i \ln \det \left[i \partial\!\!\!/ - \frac{1}{2}(1-\gamma^5)\Delta - \frac{1}{2}(1+\gamma^5)\Delta^* \right]$$

Stationary points $\Rightarrow$ gap equation : $0 = \frac{\delta S_{\text{eff}}}{\delta \Delta^*}$

Gap equation for *static* condensates :

$$\Delta(x) = -i N_f g^2 \, \text{tr}_{D,E} \left[\gamma^0 \left(\mathbf{1} + \gamma^5 \right) R(x;E) \right]$$

Coincident Green's function: $R(x;E) \equiv \langle x | \frac{1}{H-E} | x \rangle$

Gap Equation Approach

Grand potential:

$$\Psi[\Delta(x)] = \frac{1}{2g^2 N_f L} \int_0^L dx |\Delta|^2 - \frac{1}{\beta} \int dE\, \rho(E) \ln\left(1 + e^{-\beta(E-\mu)}\right)$$

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Gap equation for *static* condensates :

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Coincident Green's function: $R(x;E) \equiv \langle x | \frac{1}{H-E} | x \rangle$

Density of states : $\rho(x;E) = \frac{1}{\pi} \mathrm{Im}\, \mathrm{tr}_D \left(R(x;E+i\epsilon)\right)$

Solving the Gap Equation

Ansatz :

$$R(x;E) = \mathcal{N}(E)\begin{pmatrix} a(E)+|\Delta|^2 & b(E)\,\Delta - i\Delta' \\ b(E)\,\Delta^* + i\Delta^{*\prime} & a(E)+|\Delta|^2 \end{pmatrix}$$

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Eilenberger equation:

$$\frac{\partial}{\partial x} R(x;E)\sigma_3 = i \left[\begin{pmatrix} \text{E} & \text{-}\Delta(x) \\ \Delta^*(x) & \text{-E} \end{pmatrix}, R\sigma_3 \right]$$

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Eilenberger equation $\rightarrow$ Nonlinear Schrödinger equation:

$$\Delta'' - 2|\Delta|^2\,\Delta + i\,(b(E) - 2E)\,\Delta' - 2\,(a(E) - E\,b(E)) = 0$$

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Functional gap equation

Solving the Gap Equation

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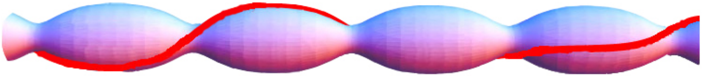
Functional gap equation $\Rightarrow$ Ordinary differential equation

Twisted kink crystal condensate (GB & Dunne, 2008)
The most general solution
4 parameters : λ, ν, θ, q

$$\Delta(x) = \lambda \frac{\sigma(\lambda x + i\mathbf{K}' - i\theta/2)}{\sigma(\lambda x + i\mathbf{K}')\sigma(i\theta/2)} e^{2iqx}$$



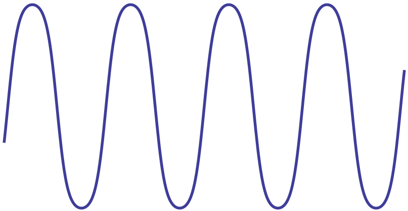
Chiral spiral (Schön,Thies, 1999)
($\theta = 0, 4\mathbf{K}'$) 2 parameters : λ, q

$$\Delta(x) \quad = \quad \lambda e^{2iqx}$$

Solutions of the NLSE Equation

Real kink crystal (Thies,Urlichs, 2005)
$(\theta = 2\mathbf{K}', q = 0)$ 2 parameters : λ, ν

$$\Delta(x) \quad = \quad \lambda\sqrt{\nu}\,\mathrm{sn}\,(\lambda x;\nu)$$



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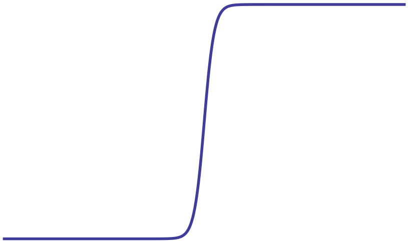

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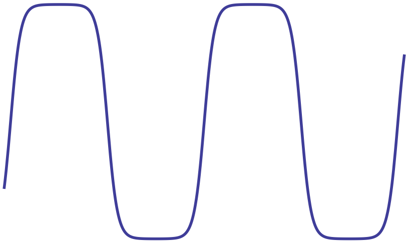
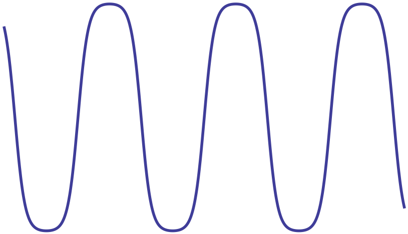
$$\Delta(x) = \lambda\sqrt{\nu}\,\mathrm{sn}\,(\lambda x; \nu)$$



Solutions of the NLSE Equation

Real kink crystal (Thies,Urlichs, 2005)
$(\theta = 2\mathbf{K}', q = 0)$ 2 parameters : λ, ν

$$\Delta(x) \quad = \quad \lambda\sqrt{\nu}\,\mathrm{sn}\,(\lambda x; \nu)$$



Solutions of the NLSE Equation

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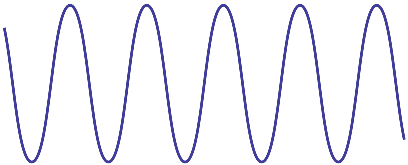

Solutions of the NLSE Equation

Real kink crystal (Thies,Urlichs, 2005)
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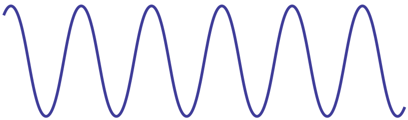


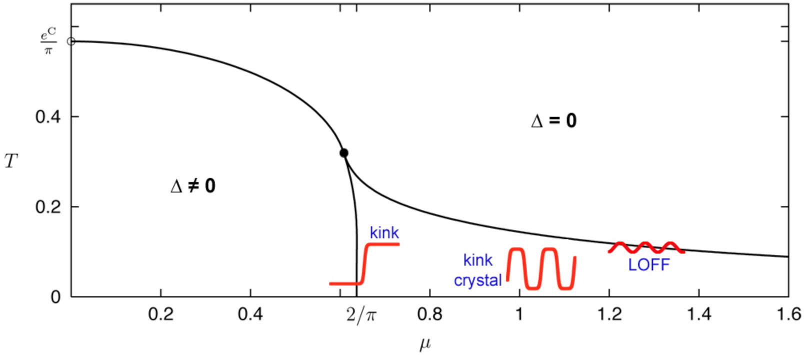
Real kink crystal (Thies,Urlichs, 2005)
$(\theta = 2\mathbf{K}', q = 0)$ 2 parameters : λ, ν

$$\Delta(x) = \lambda\sqrt{\nu}\,\mathrm{sn}\,(\lambda x; \nu)$$

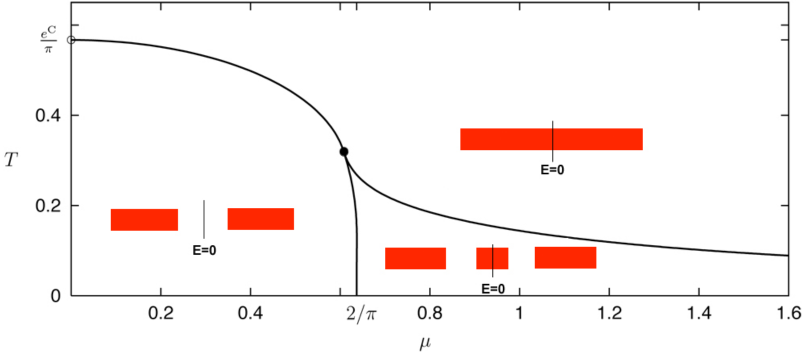
Outline

- Introduction to Gross-Neveu models
- Gap equation and inhomogeneous phases
- Phase diagrams
- Ginzburg-Landau expansion
- Connection to string theory

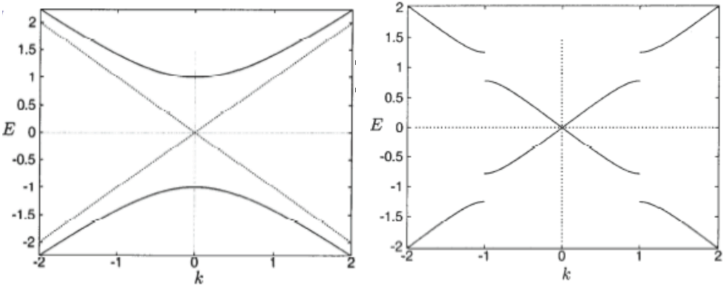
GN_2 Phase Diagram



Energy Spectrum:

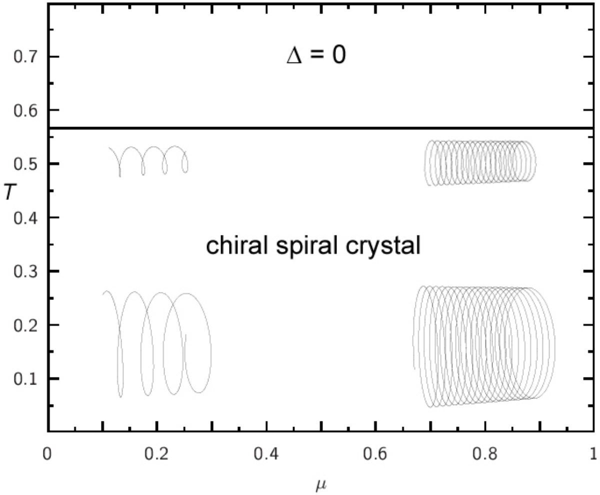


Lowering the free energy by opening a gap around the Fermi surface:



NJL_2 Phase Diagram

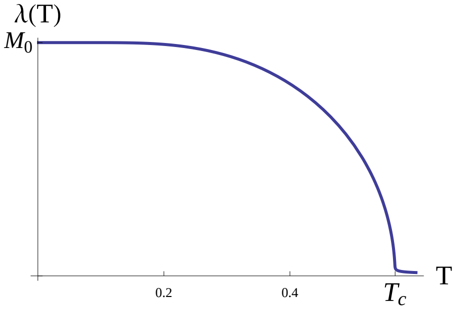
$$\Delta(x) = \lambda(T)e^{2i\mu x}$$



NJL_2 Phase Diagram

$$\Delta(x) = \lambda(T) e^{2i\mu x}$$

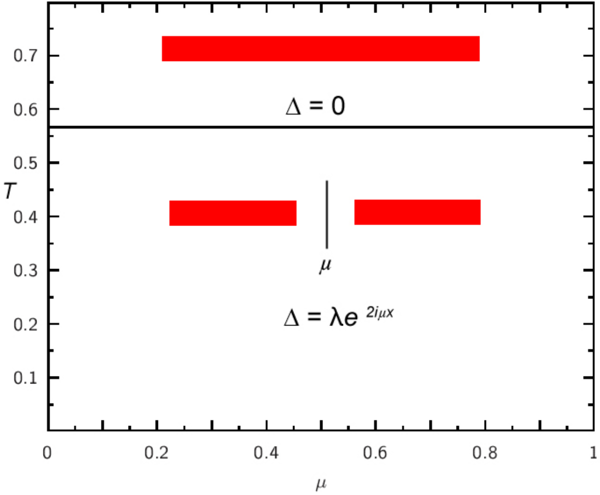
- Thermal mass scale:



- Constant charge density: $\langle \psi^\dagger \psi \rangle = \frac{\mu}{\pi}$

NJL_2 Phase Diagram

Energy Spectrum:



Due to the *continuous* chiral symmetry, the Dirac-BgD equation;

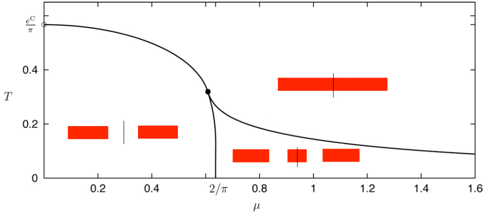
$$\mathrm{H}=\begin{pmatrix} -i\frac{d}{dx} & \Delta(x) \\ \Delta^*(x) & i\frac{d}{dx} \end{pmatrix}\psi = E\psi$$

is invariant under:

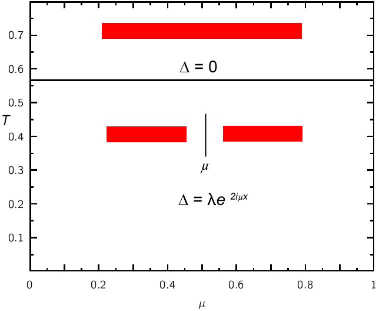
$$\Delta(x) \to \Delta(x)\, e^{2iqx} \qquad \psi(x) \to e^{iqx\gamma_5}\psi(x)$$

$$E \quad \to \quad E+q$$

Peierls instability $\Rightarrow$ q=μ

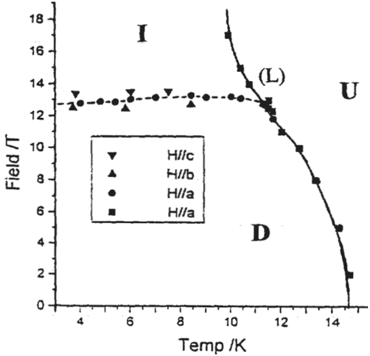


Discrete chiral symmetry→ symmetric energy spectrum + Peierls instability=1 or 2 gaps

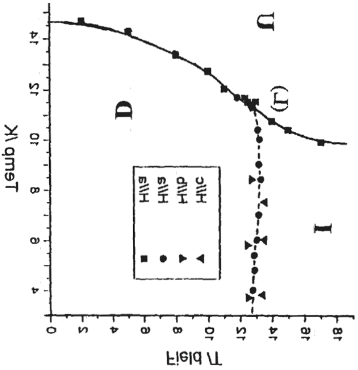


Continuous chiral symmetry→ offset in energy spectrum + Peierls instability=1 gap around μ

Inorganic spin Peierls compound $CuGeO_3$ (Boucher, Reynauld, 1996, Hofmann, 2010)



Inorganic spin Peierls compound $CuGeO_3$ (Boucher, Reynauld, 1996, Hofmann, 2010)



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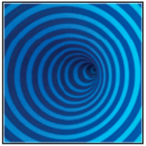
Quark gluon solenoid



Chiral Magnetic Spirals

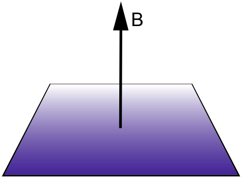
Gökçe Başar, Gerald V. Dunne, and Dmitri E. Kharzeev

Phys. Rev. Lett. 104, 232301 (Published June 7, 2010)

In heavy ion collisions (GB, Dunne, Kharzeev,2010)

Chiral Magnetic Spiral

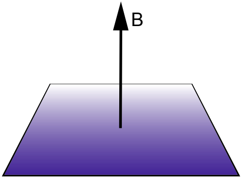
- Magnetic catalysis



- Topological charge fluctuations

Chiral Magnetic Spiral

- Magnetic catalysis



$$(\vec{p} - e\vec{A})^2 \rightarrow p_{\|}^2 + (\vec{p}_{\perp} - e\vec{A})^2$$

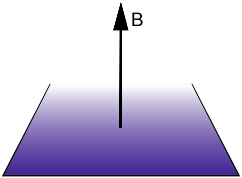
$\Downarrow$

Landau levels

- Topological charge fluctuations

Chiral Magnetic Spiral

- Magnetic catalysis



$$(\vec{p} - e\vec{A})^2 \rightarrow p_{\parallel}^2 + (\vec{p}_{\perp} - e\vec{A})^2$$

$\Downarrow$

Landau levels

- Topological charge fluctuations

$$\mu_R > \mu_L$$

Chiral Magnetic Spiral

$$\mu_R = -\mu_L = \mu_5$$

$$\Psi = (R_+, R_-, L_-, L_+)^T \quad \rightarrow \quad \mathcal{R} = (R_+, R_-)^T, \quad \mathcal{L} = (L_+, L_-)^T$$

$$\begin{aligned}
J^0 &= \mathcal{R}^\dagger \mathcal{R} + \mathcal{L}^\dagger \mathcal{L} \\
J^1 &= \bar{\mathcal{R}}\,\mathcal{R} - \bar{\mathcal{L}}\,\mathcal{L} \\
J^2 &= i\bar{\mathcal{R}}\,\Gamma^5 \mathcal{R} + i\bar{\mathcal{L}}\,\Gamma^5 \mathcal{L} \\
J^3 &= \bar{\mathcal{R}}\,\Gamma^z \mathcal{R} + \bar{\mathcal{L}}\,\Gamma^z \mathcal{L}
\end{aligned}$$

$$\begin{aligned}
J_5^0 &= \mathcal{R}^\dagger \mathcal{R} - \mathcal{L}^\dagger \mathcal{L} \\
J_5^1 &= \bar{\mathcal{R}}\,\mathcal{R} + \bar{\mathcal{L}}\,\mathcal{L} \\
J_5^2 &= i\bar{\mathcal{R}}\,\Gamma^5 \mathcal{R} - i\bar{\mathcal{L}}\,\Gamma^5 \mathcal{L} \\
J_5^3 &= \bar{\mathcal{R}}\,\Gamma^z \mathcal{R} - \bar{\mathcal{L}}\,\Gamma^z \mathcal{L}
\end{aligned}$$

Chiral Magnetic Spiral

$$\mu_R = -\mu_L = \mu_5$$

$$\Psi = (R_+, R_-, L_-, L_+)^T \quad \rightarrow \quad \mathcal{R} = (R_+, R_-)^T, \quad \mathcal{L} = (L_+, L_-)^T$$

$$\langle \psi_f^\dagger \psi_f \rangle = \frac{\mu_f}{\pi} \qquad \langle \bar{\psi}_f \psi_f - i\bar{\psi}_f i\Gamma^5 \psi_f \rangle \sim e^{2i\mu_f z}$$

$$\begin{aligned} J^0 &= \mathcal{R}^\dagger \mathcal{R} + \mathcal{L}^\dagger \mathcal{L} \\ J^1 &= \bar{\mathcal{R}}\,\mathcal{R} - \bar{\mathcal{L}}\,\mathcal{L} \\ J^2 &= i\bar{\mathcal{R}}\,\Gamma^5 \mathcal{R} + i\bar{\mathcal{L}}\,\Gamma^5 \mathcal{L} \\ J^3 &= \bar{\mathcal{R}}\,\Gamma^z \mathcal{R} + \bar{\mathcal{L}}\,\Gamma^z \mathcal{L} \end{aligned} \qquad \begin{aligned} J_5^0 &= \mathcal{R}^\dagger \mathcal{R} - \mathcal{L}^\dagger \mathcal{L} \\ J_5^1 &= \bar{\mathcal{R}}\,\mathcal{R} + \bar{\mathcal{L}}\,\mathcal{L} \\ J_5^2 &= i\bar{\mathcal{R}}\,\Gamma^5 \mathcal{R} - i\bar{\mathcal{L}}\,\Gamma^5 \mathcal{L} \\ J_5^3 &= \bar{\mathcal{R}}\,\Gamma^z \mathcal{R} - \bar{\mathcal{L}}\,\Gamma^z \mathcal{L} \end{aligned}$$

Chiral Magnetic Spiral

$$\mu_R = -\mu_L = \mu_5$$

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$$\langle J^0 \rangle = 0$$
$$\langle J^1 \rangle \sim \cos(2\mu_5 z)$$
$$\langle J^2 \rangle \sim \sin(2\mu_5 z)$$
$$\langle J^3 \rangle = \frac{eB}{2\pi}\frac{\mu_5}{\pi}$$

$$\langle J_5^0 \rangle = \frac{eB}{2\pi}\frac{\mu_5}{\pi}$$
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Chiral Magnetic Spiral

$$\mu_R = -\mu_L = \mu_5$$

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$$\langle J^0 \rangle = 0 \qquad \langle J_5^0 \rangle = \frac{eB}{2\pi}\frac{\mu_5}{\pi}$$
$$\langle J^1 \rangle \sim \cos(2\mu_5 z) \qquad \langle J_5^1 \rangle \sim \cos(2\mu_5 z)$$
$$\langle J^2 \rangle \sim \sin(2\mu_5 z) \qquad \langle J_5^2 \rangle \sim \sin(2\mu_5 z)$$
$$\langle J^3 \rangle = \frac{eB}{2\pi}\frac{\mu_5}{\pi} \qquad \langle J_5^3 \rangle = 0$$

$\Uparrow$

Chiral Magnetic Effect

Chiral Magnetic Spiral

$$\mu_R = -\mu_L = \mu_5$$

$$\Psi = (R_+, R_-, L_-, L_+)^T \quad \rightarrow \quad \mathcal{R} = (R_+, R_-)^T, \quad \mathcal{L} = (L_+, L_-)^T$$

$$\langle \psi_f^\dagger \psi_f \rangle = \frac{\mu_f}{\pi} \qquad \langle \bar{\psi}_f \psi_f - i\bar{\psi}_f i\Gamma^5 \psi_f \rangle \sim e^{2i\mu_f z}$$

$$\langle J^0 \rangle = 0 \qquad \langle J^0_5 \rangle = \frac{eB}{2\pi}\frac{\mu_5}{\pi}$$

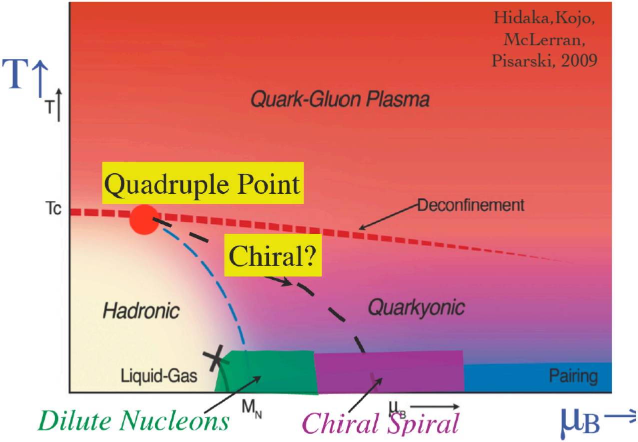
$$\langle J^1 \rangle \sim \cos(2\mu_5 z) \qquad \langle J^1_5 \rangle \sim \cos(2\mu_5 z)$$

$$\langle J^2 \rangle \sim \sin(2\mu_5 z) \qquad \langle J^2_5 \rangle \sim \sin(2\mu_5 z)$$

$$\langle J^3 \rangle = \frac{eB}{2\pi}\frac{\mu_5}{\pi} \qquad \langle J^3_5 \rangle = 0$$

Chiral Magnetic Spiral

Quarkyonic Chiral Spiral



In cold, dense QCD, "quarkyonic chiral spiral"
(Hidaka,Kojo,McLerran,Pisarski,2009)

Outline

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- Ginzburg-Landau expansion
- Connection to string theory

Ginzburg Landau Expansion

$$\begin{aligned}\Psi_{NJL} &= \alpha_0 + \alpha_2|\Delta|^2 + \alpha_3 \mathrm{Im}\left(\Delta\Delta'^*\right) + \alpha_4(|\Delta|^4 + |\Delta'|^2) \\ &+\alpha_5 \mathrm{Im}\left((\Delta'' - 3|\Delta|^2\Delta)\Delta'^*\right) \\ &+\alpha_6(2|\Delta|^6 + 8|\Delta|^2|\Delta'|^2 + 2\mathrm{Re}\Delta'^2\Delta^{*2} + |\Delta''|^2) + \ldots\end{aligned}$$

Ginzburg Landau Expansion

$$
\begin{aligned}
\Psi_{NJL} \quad = \quad & \alpha_0 + \alpha_2 |\Delta|^2 + \alpha_3 \mathrm{Im}\left(\Delta \Delta'^*\right) + \alpha_4 (|\Delta|^4 + |\Delta'|^2) \\
& + \alpha_5 \mathrm{Im}\left((\Delta'' - 3|\Delta|^2 \Delta)\Delta'^*\right) \\
& + \alpha_6 (2|\Delta|^6 + 8|\Delta|^2 |\Delta'|^2 + 2\mathrm{Re}\Delta'^2 \Delta^{*2} + |\Delta''|^2) + \ldots
\end{aligned}
$$

$$
\Psi_{GN} \quad = \quad \alpha_0 + \alpha_2 \, S^2 + \alpha_4 (S^4 + S'^2) + \alpha_6 (2S^6 + 10 S^2 \, S'^{\,2} + S''^{\,2}) + \ldots
$$

Ginzburg Landau Expansion

$$\begin{aligned}\Psi_{NJL} &= \alpha_0 + \alpha_2|\Delta|^2 + \alpha_3 \mathrm{Im}\left(\Delta\Delta'^*\right) + \alpha_4(|\Delta|^4 + |\Delta'|^2) \\ &+\alpha_5 \mathrm{Im}\left((\Delta'' - 3|\Delta|^2\Delta)\Delta'^*\right) \\ &+\alpha_6(2|\Delta|^6 + 8|\Delta|^2|\Delta'|^2 + 2\mathrm{Re}\Delta'^2\Delta^{*2} + |\Delta''|^2) + \dots\end{aligned}$$

$\Uparrow$

$c_n s$: Conserved quantities of AKNS hierarchy

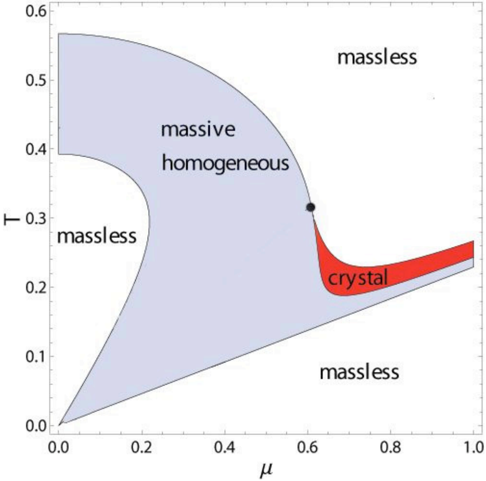
$$\Psi_{GN} = \alpha_0 + \alpha_2\, S^2 + \alpha_4(S^4 + S'^2) + \alpha_6(2S^6 + 10S^2\, S'^{\,2} + S''^{\,2}) + \dots$$

$\Uparrow$

$c_{2n} s$: Conserved quantities of mKdV hierarchy

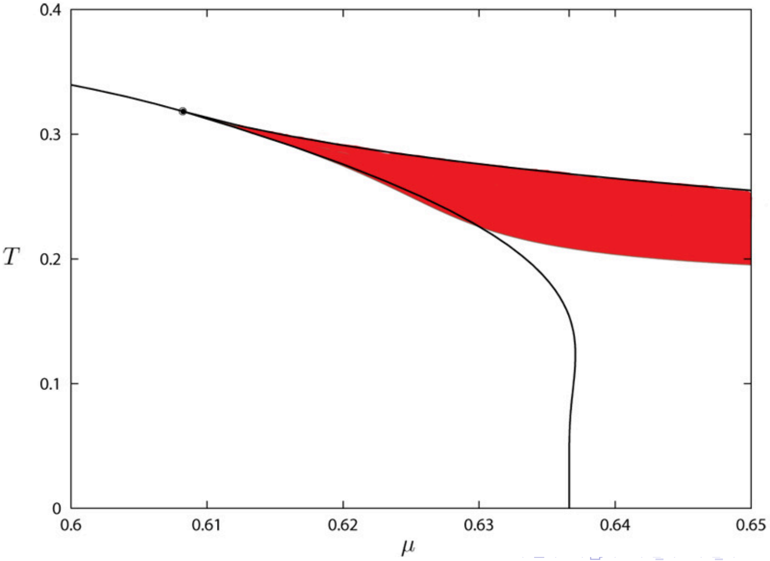
Ginzburg Landau Expansion (GN_2)

$$\Psi = \alpha_0 + \alpha_2 S^2 + \alpha_4 (S^4 + S'^2) + \alpha_6 (2S^6 + 10 S^2 S'^2 + S''^2) + \ldots$$



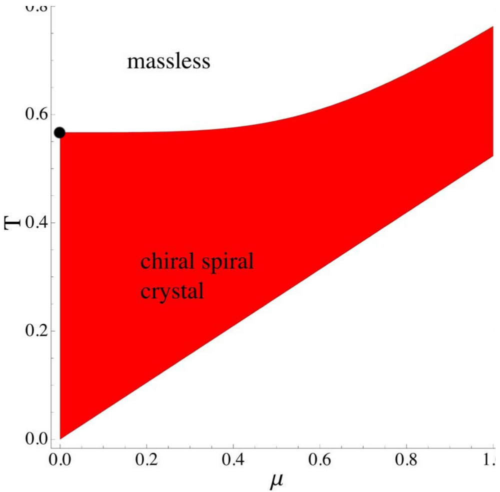
Ginzburg Landau Expansion (GN_2)

$$\Psi = \alpha_0 + \alpha_2 S^2 + \alpha_4 (S^4 + S'^2) + \alpha_6 (2S^6 + 10S^2 S'^2 + S''^2) + \dots$$



Ginzburg Landau Expansion (NJL_2)

$$\Psi = \alpha_0 + \alpha_2|\Delta|^2 + \alpha_3 \mathrm{Im}\,(\Delta\Delta'^*) + \alpha_4(|\Delta|^4 + |\Delta'|^2) + \dots$$



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In the Hartree-Fock picture:

$$\bar{\psi}_k \psi_k(x) \quad = \quad f(k) S(x) \qquad \text{for } \textit{each} \text{ mode}$$

$$\mathrm{H}\psi_k = \begin{pmatrix} -i\frac{d}{dx} & l\bar{\psi}_k\psi_k(x) \\ l\bar{\psi}_k\psi_k(x) & i\frac{d}{dx} \end{pmatrix} \psi_k = E\psi_k$$

$$(i\partial\!\!\!/ - l\bar{\psi}_k\psi_k)\psi_k = 0 \qquad \psi_k = \psi_k(x)e^{-iEt} \text{ (static solutions)}$$

Nonlinear Dirac equation

In the Hartree-Fock picture:

$$< \bar{\psi}\psi(x) >= \sum_k f(k)S(x) = -1/g^2 S(x)$$

$$\mathrm{H}\psi_k = \begin{pmatrix} -i\frac{d}{dx} & l\bar{\psi}_k\psi_k(x) \\ l\bar{\psi}_k\psi_k(x) & i\frac{d}{dx} \end{pmatrix} \psi_k = E\psi_k$$

$$(i\partial\!\!\!/ - l\bar{\psi}_k\psi_k)\psi_k = 0 \qquad \psi_k = \psi_k(x)e^{-iEt} \text{ (static solutions)}$$

Nonlinear Dirac equation

Time dependent solutions?

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$$(i\partial\!\!\!/ - l\bar{\psi}_k\psi_k)\psi_k = 0$$

$$\bar{\psi}_k\psi_k(x,t) = f(k)S(x,t)$$

Time dependent solutions?

$$(i\partial\!\!\!/ - l\bar{\psi}_k\psi_k)\psi_k = 0$$

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- Boosted kink (crystal):

$S(x) \to S(\frac{x-vt}{\sqrt{1-v^2}})$

Time dependent solutions?

$$(i\partial\!\!\!/ - l\bar{\psi}_k\psi_k)\psi_k = 0$$

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- Boosted kink (crystal):

$$S(x) \to S(\tfrac{x-vt}{\sqrt{1-v^2}})$$

- Kink-antikink scattering:

$$S(x,t) = \frac{v\cosh(2x/\sqrt{1-v^2}) - \cosh(2vt/\sqrt{1-v^2})}{v\cosh(2x/\sqrt{1-v^2}) + \cosh(2vt/\sqrt{1-v^2})}$$

Time dependent solutions?

$$(i\partial\!\!\!/ - l\bar{\psi}_k\psi_k)\psi_k = 0$$

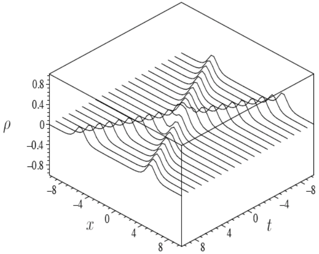
$$\bar{\psi}_k\psi_k(x,t) = f(k)S(x,t)$$

- Boosted kink (crystal):

$$S(x) \rightarrow S\left(\frac{x-vt}{\sqrt{1-v^2}}\right)$$

- Kink-antikink scattering:

$$S(x,t) = \frac{v\cosh(2x/\sqrt{1-v^2}) - \cosh(2vt/\sqrt{1-v^2})}{v\cosh(2x/\sqrt{1-v^2}) + \cosh(2vt/\sqrt{1-v^2})}$$



- Multi kink-antikink scattering

Time dependent solutions

$$(i\partial\!\!\!/ - l\bar{\psi}_k\psi_k)\psi_k = 0$$

Time dependent solutions

$$(i\partial\!\!\!/ - l\bar{\psi}_k\psi_k)\psi_k = 0$$

Self-consistency: $\bar{\psi}_k\psi_k(x,t) = f(k)S(x,t)$

Time dependent solutions

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Gap equation: $S(x,t) = \frac{\delta}{\delta S(x,t)}\mathrm{tr}\ln(i\partial\!\!\!/ - S(x,t))$

Time dependent solutions

$$(i\partial\!\!\!/ - l\bar{\psi}_k\psi_k)\psi_k = 0$$

Self-consistency: $\bar{\psi}_k\psi_k(x,t) = f(k)S(x,t)$

Gap equation: $S(x,t) = \frac{\delta}{\delta S(x,t)} \mathrm{tr}\ln(i\partial\!\!\!/ - S(x,t))$

$S = e^{\theta/2} \Rightarrow \partial_\mu\partial^\mu\theta + 4\,\mathrm{sinh}\theta = 0$ Sinh-Gordon equation

(Neveu, Papanicolaou,1978 Klotzek, Thies, 2010)

Nonlinear Dirac Equation

Static 2D solutions of GN_3

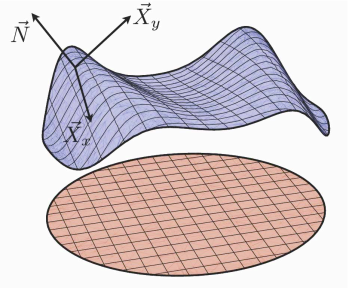
$$(i\partial\!\!\!/ - l\bar{\psi}_k\psi_k)\psi_k = 0$$

Self-consistency: $\bar{\psi}_k\psi_k(x,y) = f(k)S(x,y)$

Gap equation: $S(x,y) = \frac{\delta}{\delta S(x,y)}\operatorname{tr}\ln(i\partial\!\!\!/ - S(x,y))$

A different perspective on Gross-Neveu gap equation

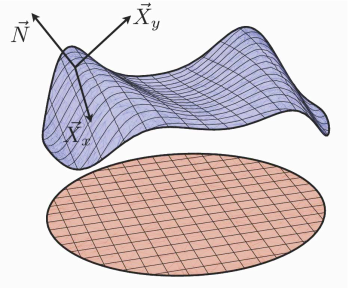
$$\vec{X}(x,t) : \Sigma \subset \mathbb{R}^{1,1} \mapsto \mathbb{R}^{1,2}$$



$$ds^2 = f^2(-dt^2 + dx^2) = f^2\, dx_- dx_+$$

Differential Geometry of Surface Embedding

$\vec{X}(x,t) : \Sigma \subset \mathbb{R}^{1,1} \mapsto \mathbb{R}^{1,2}$



$$ds^2 = f^2(-dt^2 + dx^2) = f^2\, dx_- dx_+$$

evolution of the basis frame:

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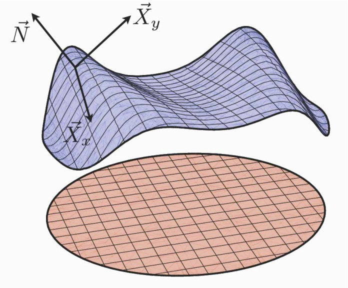
Gauss-Codazzi equations

Differential Geometry of Surface Embedding

$$\vec{X}(x,t) : \Sigma \subset \mathbb{R}^{1,1} \mapsto \mathbb{R}^{1,2}$$



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Gauss-Codazzi equations

Constant mean curvature (H) $\Leftrightarrow$ Sinh-Gordon equation

$$\partial_\mu \partial^\mu \theta + 4 \sinh\theta = 0 \quad (f^2 = e^\theta)$$

Spinor Representation of Surfaces

(Weierstrass, Hopf, Taimanov, Konopelchenko, Bobenko..., 1866-2010)

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$$\vec{X}_+, \quad \vec{X}_-, \quad \vec{N} \quad \Leftrightarrow \quad \mathrm{SU}(1,1) \text{ spinors } \psi$$

"Spinor - Surface Dictionary"

$$\begin{aligned}
\text{Dirac equation} \quad &: \quad (i\partial\!\!\!/ - S)\psi = 0 \\
\text{induced metric factor} \quad &: \quad f = \bar{\psi}\psi \\
\text{mean curvature} \quad &: \quad S = H\,(\bar{\psi}\psi) \\
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Self consistent solutions of GN_2 that satisfy $(i\partial\!\!\!/ - l\bar{\psi}\psi)\psi = 0$

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Minimal surfaces (zero mean curvature) in AdS_3

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(GB, Dunne, arXiv:1011.3835)
(Klotzek, Thies, arxiv:1006.0324)

Static Solutions in GN_2

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Curves in $\mathbb{R}^{1,2}$

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Deformations of the curves $\Leftrightarrow$ mKdV hierarchy

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Thermodynamics of GN_2

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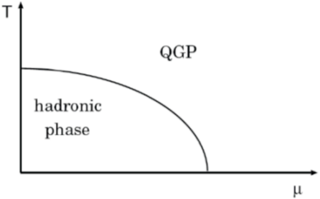
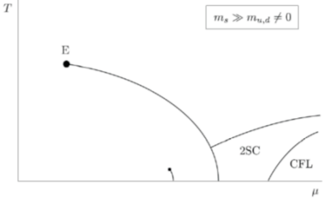
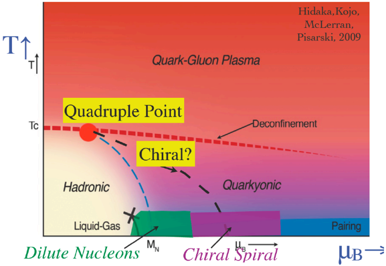
Thermodynamics of GN_3 ??

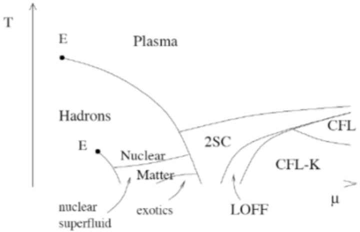
Conclusions

- Analytical solution of the inhomogeneous gap equation GN_2, NJL_2
- Crystalline phases
- Peierls instability + chiral symmetry breaking
- Integrable hierarchies $\Leftrightarrow$ Ginzburg-Landau expansion
- Solutions of NLDE of GN_2 $\Leftrightarrow$ classical strings in AdS_3









"There is a theory which states that if ever anyone discovers exactly what the Universe is for and why it is here, it will instantly disappear and be replaced by something even more bizarre and inexplicable. There is another theory which states that this has already happened."

Douglas Adams