- **Standing waves**
- **Resonances**

Background reading and web activities:

- *AAP Ch 2, sec 2.1, second half (starting from p 27)*
- *"…Standing Wave Diagrams 1" (Zona Land)*
- *"Wave Interference 2" (Zona Land)*

0. Today's objectives

After today's class, you should be able to:

- Explain what is shown in a standing-wave diagram, using the terms *envelope, node, antinode*
- Define the term *resonance* and explain conceptually how it relates to a standing wave
- Identify the boundary conditions...
  - on a string
  - in a tube
- Calculate string or tube resonance wavelengths

0. Mindset

In the discussion this week and next, we will **emphasize the concepts** before introducing the formulas.

You can remember (or rederive) the formulas more easily if you understand the concepts!

Upcoming labs will give you a chance to practice **applying** the concepts (and the formulas).

1. Standing waves

- Why does a **real-world object** vibrate in a way that produces **complex waves**?
  - Objects have **multiple modes of vibration**
  - This is because multiple waves “fit” an object

1. Standing waves

- Why does a **real-world object** vibrate in a way that produces **complex waves**?
  - Objects have **multiple modes of vibration**
  - This is because multiple waves “fit” an object
- “Violin string” demo, Zona Land (seen last week)
  - String vibrates with “one loop”, “two loops”, etc.
  - Each vibration pattern → **one sine wave**
  - When **multiple** vibration patterns are present, their sine waves are **added together**

→ *What do we get when sine waves are added?*

1. Standing waves

- What "size" waves will "fit" a vibrating object?
  - A wave "fits" if it forms a **standing wave**: an oscillating pattern, stable in space over time
  - A wave that "fits" an object is called a **resonance**

- What does a standing wave look like?
  See web demo "Wave Interference 2" (Zona Land)

  Which is a standing wave?
  - Setting "Sinusoid 6"
  - Setting "Sinusoid 8"

1. Standing waves

- What "size" waves will "fit" a vibrating object?
  - A wave "fits" if it forms a **standing wave**: an oscillating pattern, stable in space over time
  - A wave that "fits" an object is called a **resonance**

- What does a standing wave look like?
  See web demo "Wave Interference 2" (Zona Land)

  Which is a standing wave?
  - Setting "Sinusoid 6": creates a standing wave
  - Setting "Sinusoid 8": not a standing wave

1. Standing waves

- Standing waves arise because of **reflection** and **interference**
- We will not pursue this in detail; think of it this way:
  - When a “right-size wave” *reflects* from the edge of the object, the outgoing and returning waves *interfere* in a way that makes a standing wave (stable oscillation)
  - When a “wrong-size wave” *reflects* from the edge of the object, the outgoing and returning waves *interfere* in a way that is not a stable oscillation

1. Standing waves

- *Warning:* **Standing-wave diagrams** are graphs showing **amplitude** by **distance**
  - This is like the snapshot of the waves on the surface of a lake, showing water height at different physical locations
  - This is ***not*** like a typical waveform plot of a sound wave, which plots amplitude by **time** at a fixed location
- *This is useful!* The resonances of a tube of air (like the vocal tract!) depend on its **physical length**, which we can measure or calculate

1. Standing waves

- Some key terminology:
  - **node** — physical position on standing wave that **always has zero amplitude**

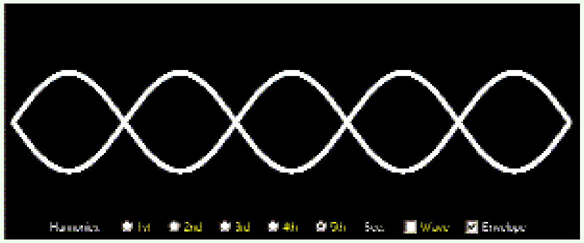
    = location in space where a wave and its reflection always *cancel each other out*
  - **antinode** — physical position on standing wave with **maximum amplitude change from zero** (oscillates between **positive & negative** extreme values)

    = location in space where a wave and its reflection *maximally reinforce each other*
- Where are the nodes and antinodes on "Sinusoid 6"?

1. Standing waves

- **Standing-wave diagrams** typically show the **envelope** of the standing wave
  - This shows the *maximum* amplitude reached at each physical position along the wave
  - See web demo "Understanding Standing Wave Diagrams 1" (Zona Land)

- Try it: Can you identify the **nodes** and **antinodes** on this standing-wave diagram?



(from the demo linked above)

2. Resonances

So far, we've considered these ideas:

- Why does a **real-world object** vibrate in a way that produces **complex waves**?
    - Objects have **multiple modes of vibration**
    - This is because **multiple waves "fit" an object**

What "size" waves will "fit" a vibrating object?

- A wave "fits" if it forms a ___________________, which is _____________________
    - A wave that "fits" an object is called a _____________

2. Resonances

So far, we've considered these ideas:

- Why does a **real-world object** vibrate in a way that produces **complex waves**?
  - Objects have **multiple modes of vibration**
  - This is because **multiple waves "fit" an object**

What "size" waves will "fit" a vibrating object?

- A wave "fits" if it forms a **standing wave**, which is an oscillating pattern, stable in space over time
  - A wave that "fits" an object is called a **resonance**

2. Resonances

- Next, we want to be able to **determine** what the **resonances** ("waves that fit") **of a particular object** actually are
  - We will talk about **strings** first (easy to visualize)
  - But our main focus will be on **air in a tube**

- *Preview: **Speech-sound analyses** that depend on resonance frequencies of **air in a tube** will include:*
  - *Vowel formants (indicate height, backness/rounding)*
  - *Consonant place of articulation (affects vowel formants)*
  - *Fricative noise spectra / stop burst spectra*
  - *Acoustic signatures of nasals and laterals*

2. Resonances

- We can model the **multiple modes of vibration** of a **string**, or of **air in a tube**
  - To do this, we determine the **wavelength** of each of the **resonances** of the system, based on the physical size of the system

(Next class:)

  - Then (for air in a tube) we can calculate the **frequency** of each of the resonances

2. Resonances

- The resonances ("waves that fit") for a string or a tube are determined by
  - its **length**
  - its **boundary conditions**

- **Boundary conditions**: For each **end** of the string or tube, is it a **node** or an **antinode**?
  - String: Is the end ***fixed*** (**node**) or ***free*** (**antinode**)?
  - Tube: Is the end ***open*** (**node** of pressure wave) or ***closed*** (**antinode** of pressure wave)?

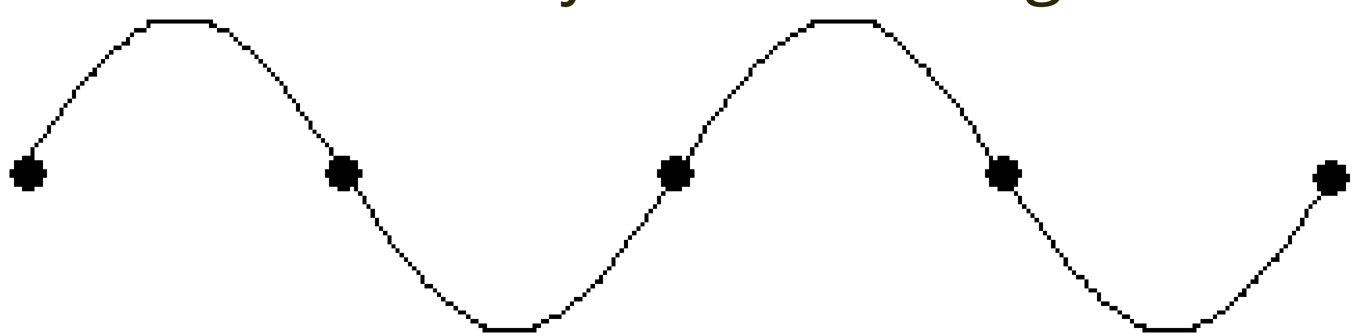
3. Boundary conditions (1): Node/node

- A **string**, fixed at both ends: **Node/node system**
  - At a point where a string is **fixed**, its displacement *can only be zero* = **node**
  - If **both ends** of the string are **fixed**, what are the "compatible waves" for this system?

3. Boundary conditions (1): Node/node

- A **string**, fixed at both ends: **Node/node system**
  - At a point where a string is **fixed**, its displacement *can only be zero* = **node**
  - If **both ends** of the string are **fixed**, what are the "compatible waves" for this system?

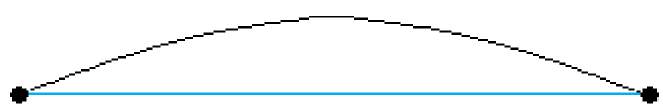
"How much wave" can fit on the string?



3. Boundary conditions (1): Node/node

- A **string**, fixed at both ends: **Node/node system**
  - At a point where a string is **fixed**, its displacement *can only be zero* = **node**
  - If **both ends** of the string are **fixed**, what are the "compatible waves" for this system?

- Here is the **first** resonance (*longest* wavelength):

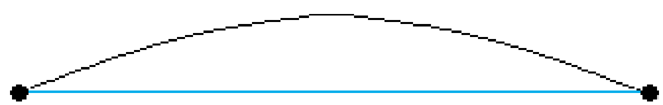


How much of the wave "fits"?

3. Boundary conditions (1): Node/node

- A **string**, fixed at both ends: **Node/node system**
  - At a point where a string is **fixed**, its displacement *can only be zero* = **node**
  - If **both ends** of the string are **fixed**, what are the "compatible waves" for this system?

- Here is the **first** resonance (*longest* wavelength):



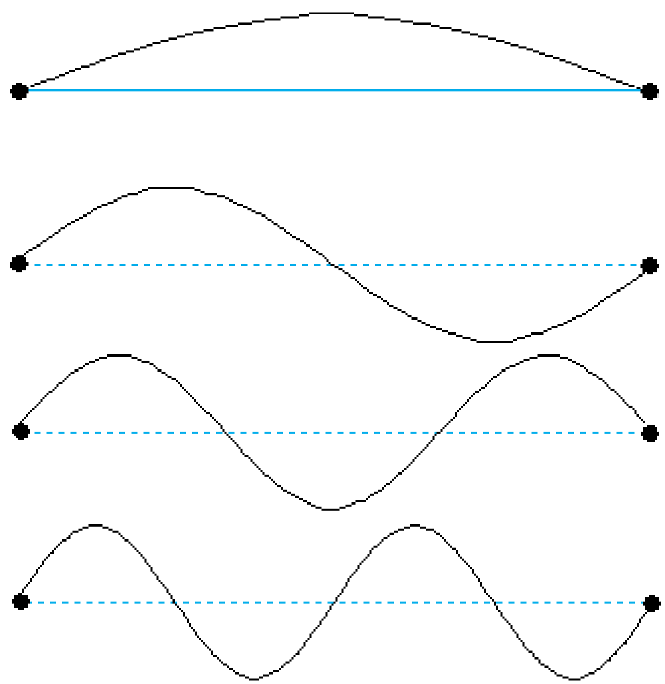
→ ***½ cycle*** of the wave "fits"

What are the rest?

3. Boundary conditions (1): Node/node

- A **string**, fixed at both ends: **Node/node system**, also called a **half-wavelength system** (why?)

- Here are the first four **resonances**



1st → ½ *cycle* of the wave “fits”

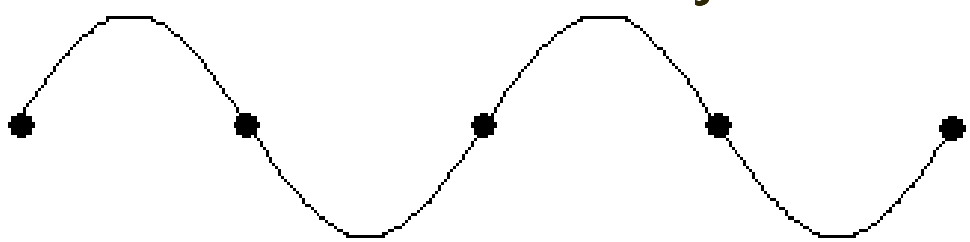
2nd → *1 cycle* of the wave “fits”

3rd → 1½ *cycles* of the wave “fit”

4th → *2 cycles* of the wave “fit”

3. Boundary conditions (1): Node/node

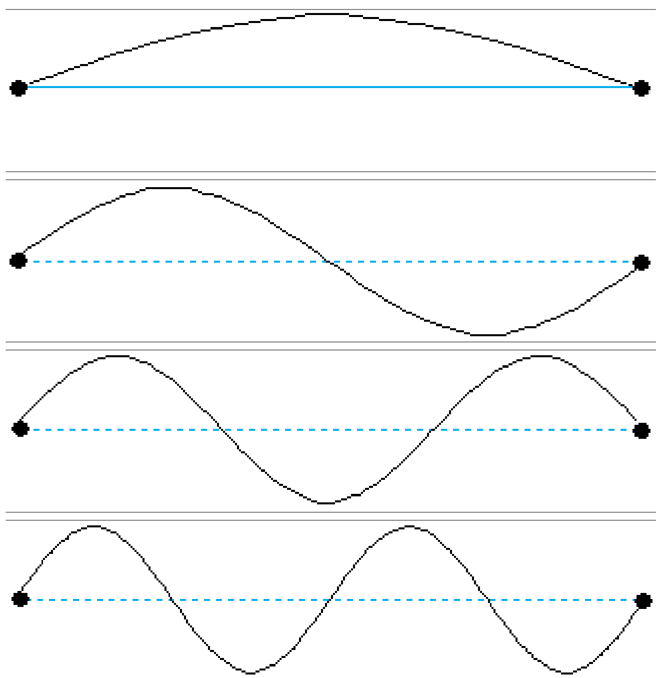
- A **tube**, open at both ends: **Node/node system**
  - At a point where a tube is **open**, the air pressure interfaces with the outside world, so the **pressure** *can only be zero* = **node**
  - If **both ends** of the tube are **open**...
    "How much wave" can fit in the tube?



- See web demo "Standing Sound Waves" for more about pressure waves in a tube and standing-wave diagrams — the **red graph** in that demo shows the **pressure** wave

3. Boundary conditions (1): Node/node

- A **tube**, open at both ends: **Node/node system**, also called a **half-wavelength system**
  - Exactly like the string case discussed above!
- Here are the first four **resonances**



1st → ½ *cycle* of the wave “fits”

2nd → *1 cycle* of the wave “fits”

3rd → *1½ cycles* of the wave “fit”

4th → *2 cycles* of the wave “fit”

3. Boundary conditions (2): Node/antinode

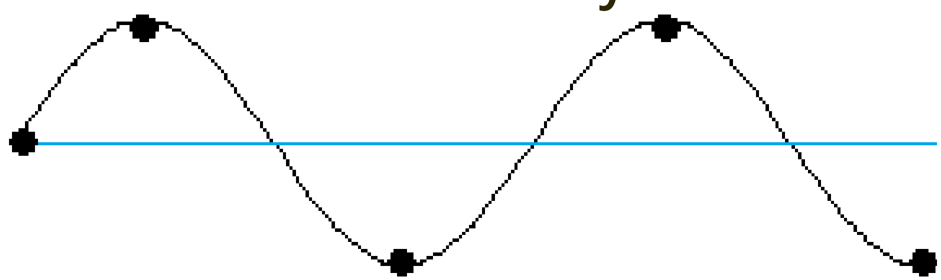
- A **tube**, open at one end and closed at the other: **Node/antinode system**
  - **Open** end: A **pressure**-wave **node** (see above)
  - **Closed** end: The reflecting waves will create a region of *maximum compression* alternating with *maximum rarefaction* → **pressure**-wave **antinode**

- *Optional:* For more about the reflection of pressure waves in tubes, see web demo "Animations of sound waves in open and closed tubes" (UNSW)
  - What happens at the edges when the wave reflects?

3. Boundary conditions (2): Node/antinode

- A **tube**, open at one end and closed at the other: **Node/antinode system**
  - **Open** end: A **pressure**-wave **node**
  - **Closed** end: A **pressure**-wave **antinode**

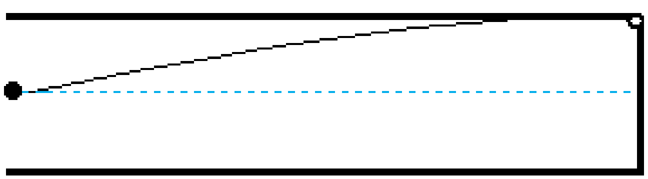
"How much wave" can fit in the tube?



3. Boundary conditions (2): Node/antinode

- A **tube**, open at one end and closed at the other: **Node/antinode system**
  - **Open** end: A pressure-wave **node**
  - **Closed** end: A pressure-wave **antinode**

- Here is the **first** resonance (*longest* wavelength):



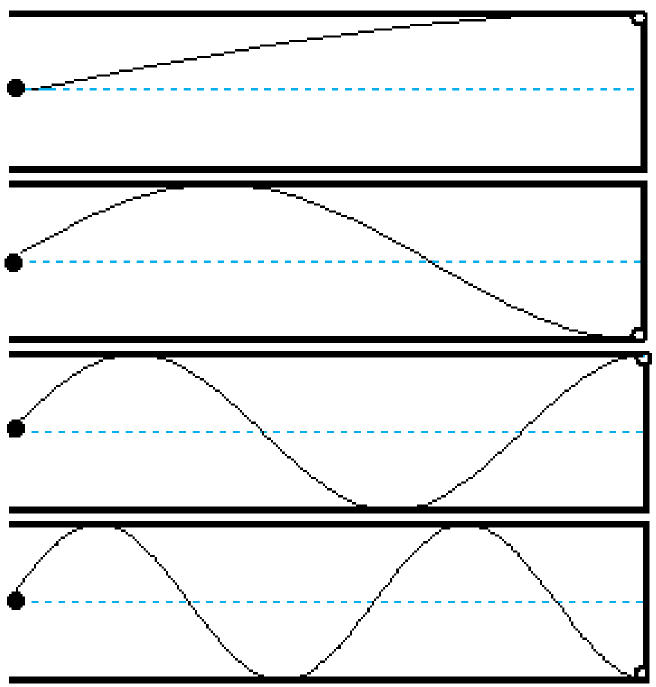
→ *¼ cycle* of the wave "fits"

What are the rest?

3. Boundary conditions (2): Node/antinode

- A **tube**, open at one end and closed at the other: **Node/antinode system**, also called a **quarter-wavelength system** (why?)

- Here are the first four **resonances**



1st → *¼ cycle* of the wave “fits”

2nd → *¾ cycle* of the wave “fits”

3rd → *1¼ cycles* of the wave “fit”

4th → *1¾ cycles* of the wave “fit”

4. Calculating resonance wavelengths

- If we know
  - the **length** of the string or tube (***L***)
  - **“how much wave” fits** on the string or tube for the ***n*th resonance**

- We can calculate the **wavelength λ_n** for the ***n*th resonance**

4. Calculating resonance wavelengths

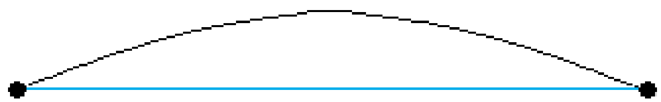
- Consider the **first resonance** of a **node/node** (half-wavelength) system

→ ½ *cycle* of the wave “fits”

- If the string or tube is length *L*,
  what is the **wavelength** of the **first** resonance (**λ_1**)?

4. Calculating resonance wavelengths

- Consider the **first resonance** of a **node/node** (half-wavelength) system



→ ½ *cycle* of the wave "fits"

- If the string or tube is length L,
  what is the **wavelength** of the **first** resonance ($\boldsymbol{\lambda_1}$)?

L fits half of the wave

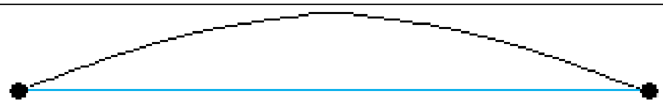
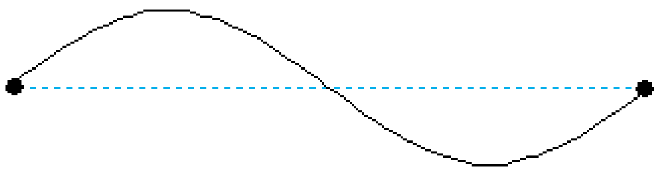
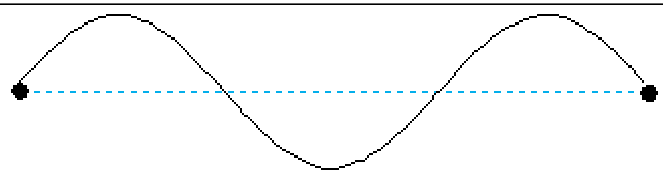
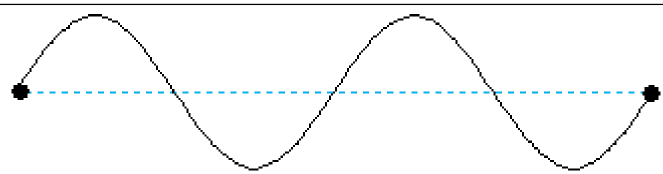
L is half as long as λ_1

$L = ½\, \lambda_1$

$$\boxed{\boldsymbol{\lambda_1 = 2L}}$$

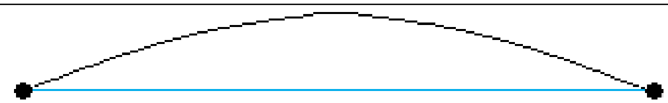
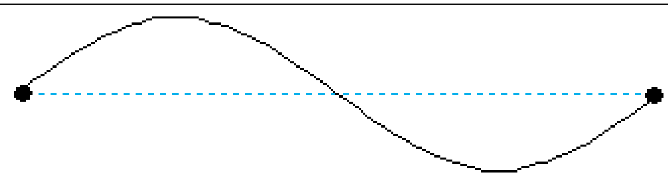
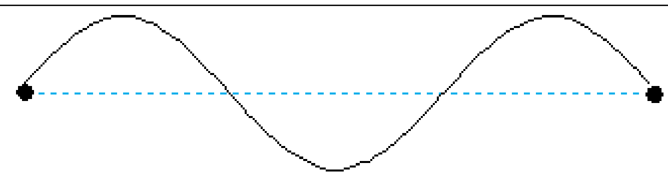
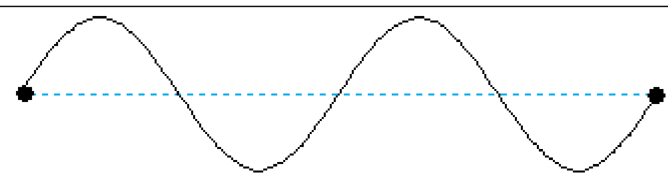
4. Calculating resonance wavelengths

- **Node/node** (half-wavelength) system
  Tube or string of length L

	$L = ½ \cdot \lambda_1$	$\lambda_1 = 2\,L$	
	$L =$ __ $\cdot\ \lambda_2$	$\lambda_2 =$	
	$L =$ __ $\cdot\ \lambda_3$	$\lambda_3 =$	
	$L =$ __ $\cdot\ \lambda_4$	$\lambda_4 =$	

4. Calculating resonance wavelengths

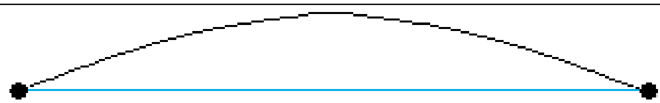
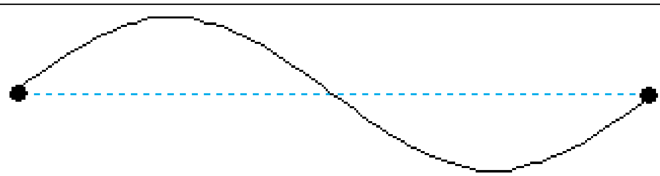
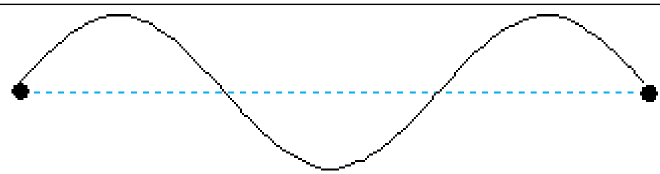
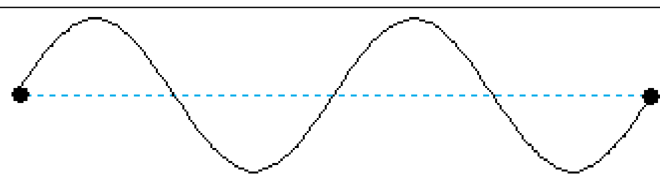
- **Node/node** (half-wavelength) system
  Tube or string of length L

	$L = ½ \cdot \lambda_1$	$\lambda_1 = 2\,L$	
	$L = 1 \cdot \lambda_2$	$\lambda_2 = L$	
	$L = 1½ \cdot \lambda_3$	$\lambda_3 = ⅔\,L$	
	$L = 2 \cdot \lambda_4$	$\lambda_4 = ½\,L$	

- ***General formula:*** ??

4. Calculating resonance wavelengths

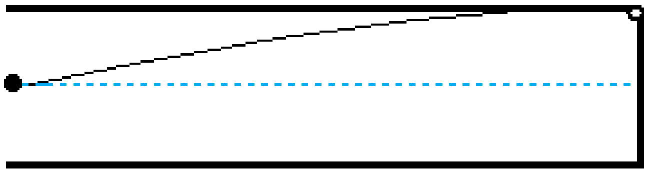
- **Node/node** (half-wavelength) system
  Tube or string of length *L*

	$L = \frac{1}{2} \cdot \lambda_1$	$\lambda_1 = 2\,L$	$\lambda_1 = (2/1) \cdot L$
	$L = 1 \cdot \lambda_2$	$\lambda_2 = L$	$\lambda_2 = (2/2) \cdot L$
	$L = 1\frac{1}{2} \cdot \lambda_3$	$\lambda_3 = \frac{2}{3}\,L$	$\lambda_3 = (2/3) \cdot L$
	$L = 2 \cdot \lambda_4$	$\lambda_4 = \frac{1}{2}\,L$	$\lambda_4 = (2/4) \cdot L$

- ***General formula:*** $\lambda_n = (2/n)\,L$ or $\lambda_n = 2L/n$

4. Calculating resonance wavelengths

- Consider the **first resonance** of a **node/antinode** (quarter-wavelength) system

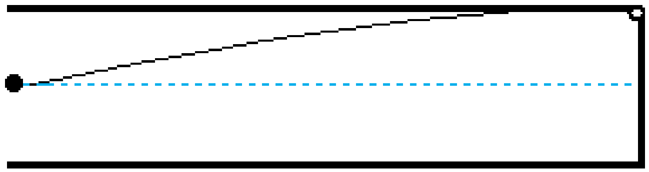


→ ¼ *cycle* of the wave "fits"

- If the tube is length L,
  what is the **wavelength** of the **first** resonance (λ_1)?

4. Calculating resonance wavelengths

- Consider the **first resonance** of a **node/antinode** (quarter-wavelength) system



→ ¼ *cycle* of the wave "fits"

- If the tube is length L,
  what is the **wavelength** of the **first** resonance ($\boldsymbol{\lambda_1}$)?

L fits one quarter of the wave

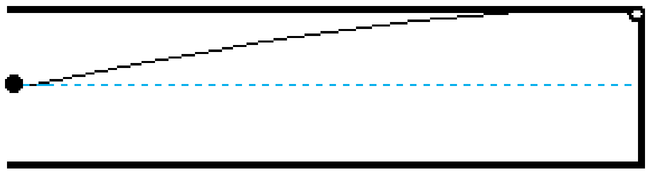
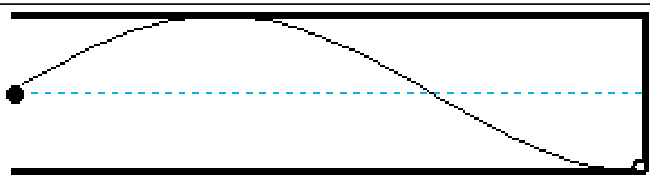
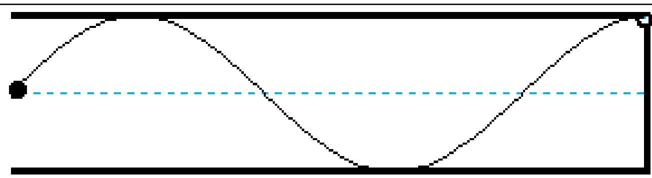
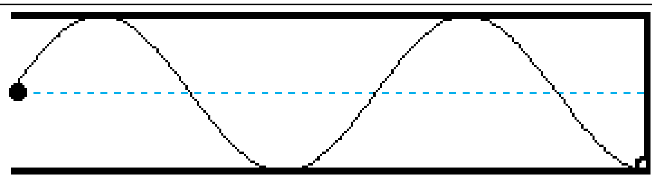
L is one quarter as long as λ_1

$L = \frac{1}{4} \lambda_1$

$\boxed{\boldsymbol{\lambda_1 = 4L}}$

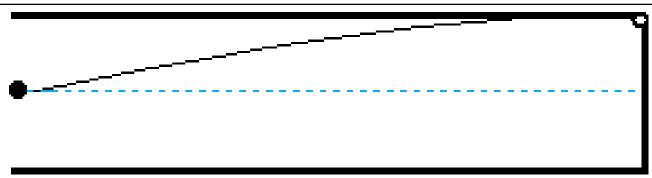
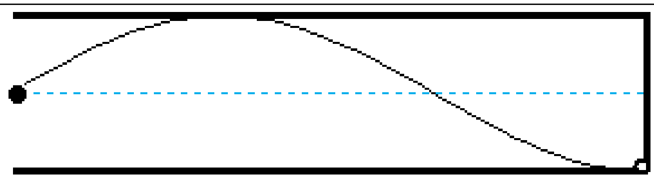
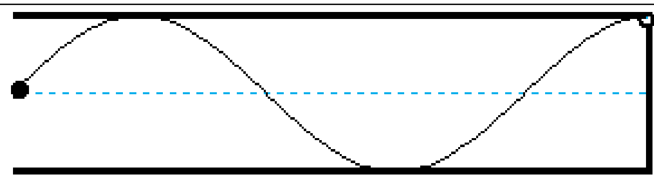
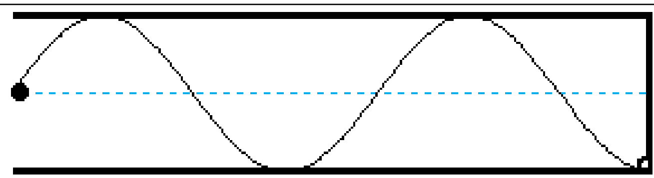
4. Calculating resonance wavelengths

- **Node/antinode** (quarter-wavelength) system
  Tube of length *L*

	$L = ¼ \cdot \lambda_1$	$\lambda_1 = 4\,L$	
	$L = \_\_ \cdot \lambda_2$	$\lambda_2 =$	
	$L = \_\_ \cdot \lambda_3$	$\lambda_3 =$	
	$L = \_\_ \cdot \lambda_4$	$\lambda_4 =$	

4. Calculating resonance wavelengths

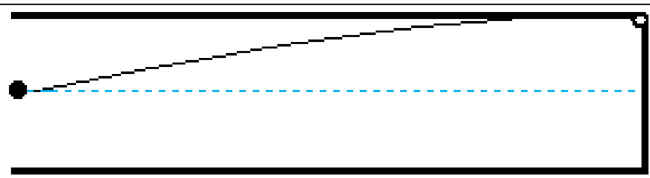
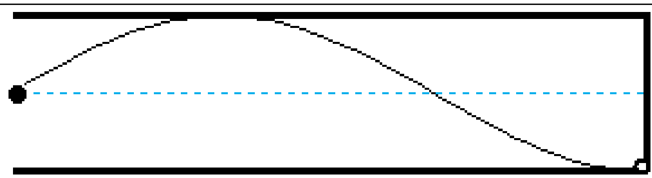
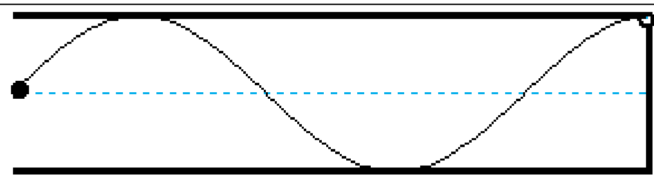
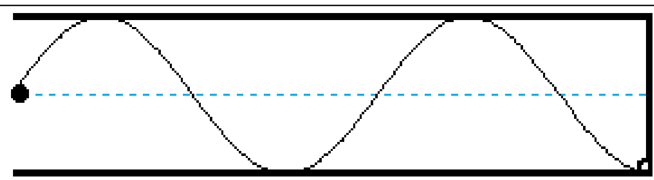
- **Node/antinode** (quarter-wavelength) system
  Tube of length L

	$L = \frac{1}{4} \cdot \lambda_1$	$\lambda_1 = 4\ L$	
	$L = \frac{3}{4} \cdot \lambda_2$	$\lambda_2 = 4/3\ L$	
	$L = 1\frac{1}{4} \cdot \lambda_3$	$\lambda_3 = 4/5\ L$	
	$L = 1\frac{3}{4} \cdot \lambda_4$	$\lambda_4 = 4/7\ L$	

- ***General formula:*** ??

4. Calculating resonance wavelengths

- **Node/antinode** (quarter-wavelength) system
  Tube of length L

	$L = \frac{1}{4} \cdot \lambda_1$	$\lambda_1 = 4\ L$	$\lambda_1 = (4/\mathbf{1}) \cdot L$
	$L = \frac{3}{4} \cdot \lambda_2$	$\lambda_2 = 4/3\ L$	$\lambda_2 = (4/\mathbf{3}) \cdot L$
	$L = 1\frac{1}{4} \cdot \lambda_3$	$\lambda_3 = 4/5\ L$	$\lambda_3 = (4/\mathbf{5}) \cdot L$
	$L = 1\frac{3}{4} \cdot \lambda_4$	$\lambda_4 = 4/7\ L$	$\lambda_4 = (4/\mathbf{7}) \cdot L$

- ***General formula:*** $\boldsymbol{\lambda_n = (4/(2n-1)) \cdot L}$ or $\boldsymbol{\lambda_n = 4L\ /\ (2n-1)}$

5. Next time

- There is one more step: The **frequencies** of the resonances are what we really want to know
  - Knowing the resonance frequencies of a tube of air (in the vocal tract) helps us **model the acoustics of speech sounds**
- Next class, we will start by talking about how to convert from wavelength to frequency
- Then you will have a chance to apply these concepts in Lab #03